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ABSTRACT

The emphasis of the activities presented in this monograph is on the mathematics done by students in grades 7 through 12 with the aid of microcomputers. The computer is used as an aid, to calculate, to motivate, to handle information, or to display data. The vital part of each activity is not the computer program listing, but the results of the program, the student's experience with the underlying mathematics, and what the student does with the results. The activities are representative and illustrative of the many ways in which computers are used to present and motivate mathematical concepts and methods, with enjoyment as a key outcome. Some are original, while some are available in the public domain. For each activity, the following format is used: general topic, specific topic, objective(s), description, uses and extensions, and listing. Student worksheets and sample runs are included for some programs. The general topics included are arithmetic, measurement, computation and problem solving, algebra, geometry, number theory, graphing, probability and statistics, computing and programming, and calculus and advanced mathematics. Program conversion suggestions and software references are appended. (MNS)

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ACTIVITIES MADE TO N-JOY

MATHEMATICS ACTIVITIES USING COMPUTERS

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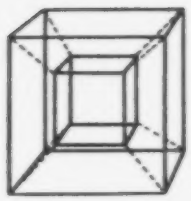
ASSOCIATION OF MATHEMATICS TEACHERS OF NEW JERSEY

MONOGRAPH #2 1985

SE 046 202

ACTIVITIES MADE TO N-JOY

MATHEMATICS ACTIVITIES USING COMPUTERS



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INTRODUCTION

This monograph is the second one published by the Association of Mathematics Teachers of New Jersey. The NCTM's "Agenda for Action" in 1980, recommended that "mathematics programs take full advantage of the power of calculators and computers at all grade levels." It is with this purpose in mind that this monograph was prepared.

The emphasis of the activities presented here is the mathematics done by the student with the aid of a microcomputer. These activities are designed for use in grades 7 through 12. However, no grade level is specified for any particular activity.

The activities and programs contained in this monograph are not meant to be definitive, commercial quality products. Rather, they are representative and illustrative of the many ways in which computers are used to present and motivate mathematical concepts and methods. Overall, the activities are made to enjoy.

Some of the activities are original and some are available in the public domain. All the activities presented here appear in the following format:

General Topic
- Specific Topic
  - Objectives
  - Description
  - Uses and Extensions
  - Listing.

Student worksheets and sample RUNs are included for some of the topics.

The General Topic areas of the activities are: Arithmetic, Measurement, Computation and Problem Solving, Algebra, Geometry, Number Theory, Graphing, Probability and Statistics, Computing and Programming, and Calculus and Advanced Mathematics.

Under each General Topic are Specific Topic areas, such as: whole number operations, polar curves, quadratic equations, random number generation, bar graphs, area problems, approximation with series, combinatorial problems, IRA's, and many more!

The computer programs contained in the activities were written by teachers for teachers. They were edited for use on most major brands of microcomputers, and they were run and debugged on an Apple IIe system. Minor changes may have to be made in the programs in order to run them on other microcomputers; a list of conversion ideas and notations have been appended. Most of the programs are short enough to be entered by students. You may wish, however, to save some of the longer programs on a disk before hand.

Again, the emphasis in each of the activities is the mathematics done by the student. The computer is used as an aid, to calculate, to motivate, to handle information, or to display data. The vital part of each activity is not the computer program listing, but the results of the program, the student's experience with the underlying mathematics, and what the student does with the results.

The Monograph could not have been written without the efforts and contributions of many individuals. I therefore express my deepest gratitude to the following authors and contributors:

Drew Antale, Ellen Brockman, Carolyn Canfield, Judy Galano, David Glatzer, John Golebiewski, Frank Gwalthney, Kathy Kopreski, Robert Mitchell, Ed Moore, Meryl Nelson, Michael O'Brien, Carolyn Rosenberg, Gloria Sanok, Joyce Scalzitti, Elaine Shaughnessy, Tom Tobiasen, and John Walter.

I also give special acknowledgement to Dr. Fran Masat, who diligently chaired this project, and to his Editorial Board: Kathy Kopreski, Ed Moore, Elaine Shaughnessy, and John Walter.

Lastly, I want to thank Glassboro State College for their cooperation and support in helping to make this Monograph a reality.

Thomas Buscemi,
President, AMTNJ

February 1, 1985
Vineland, New Jersey

NOTES TO THE TEACHER

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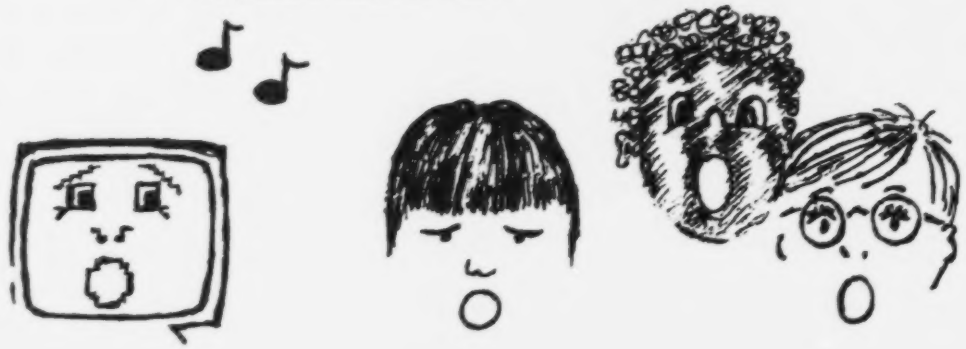

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General Topic : Arithmetic and Measurement

Specific Topic : Whole Number Operations: Addition, Subtraction and Multiplication

Objective : The student will be able to determine the answer to an addition subtraction or multiplication problem and check it with the answer provided by the computer.

Description : The activity is based on a computer program that produces two random numbers and an operation. The student responds with his or her answer, receiving a correct or in-correct response.

Uses and Extensions : Drill and practice for adding, subtracting and multiplying. The activity may be extended to more difficult problems by changing lines 105 and 110 (205 and 206, 305 and 306, 403 and 404) to N1=INT(RND(1)*100) and N2=INT(RND(1)*100) respectively. See the activity "Whole Number Operations: Drill and Practice..." under Arithmetic.

```
LIST

10  PRINT "SOLVE 20 COMPUTATION PROBLEMS ": PRINT
11  PRINT "INPUT THE NUMBER OF THE TYPE OF PROBLEMS YOU WOULD LIKE TO ATT
     EMPT": PRINT
12  PRINT "1) ADD      2) SUBTRACT      3) MULTIPLY     4) A MIXTURE": PRINT

13  INPUT A:C = 0
35  ON A GOTO 100,200,300,400
38  GOTO 11
100  REM      ADDING
101  FOR X = 1 TO 20
105  LET N1 =  INT ( RND (1) * 11)
110  LET N2 =  INT ( RND (1) * 10)
115  LET Y = N1 + N2
120  PRINT "WHAT IS THE ANSWER TO ";N1;" + ";N2
125  INPUT S
130  IF S = Y THEN  PRINT "GOOD WORK !!":C = C + 1: GOTO 145
135  PRINT "SORRY, BUT THE CORRECT ANSWER IS ";Y
145  NEXT X
150  GOTO 600
200  REM        SUBTRACTION
201  FOR X = 1 TO 20
205  LET N1 =  INT ( RND (1) * 11)
206  LET N2 =  INT ( RND (1) * 10)
207  IF N1 > N2 THEN  GOTO 210
208  LET Y = N1: LET N1 = N2: LET N2 = Y
210  LET Y = N1 - N2
220  PRINT "WHAT IS THE ANSWER TO ";N1;" - ";N2
225  INPUT S
230  IF S = Y THEN  PRINT "GOOD WORK !!":C = C + 1: GOTO 245
235  PRINT "SORRY, BUT THE CORRECT ANSWER IS ";Y
245  NEXT X
250  GOTO 600
  .
  .
  .
```

```
    :
    :
    .
300  REM      MULTIPLICATION
301  FOR X = 1 TO 20
305  LET N1 =  INT ( RND (1) * 11)
306  LET N2 =  INT ( RND (1) * 10)
310  LET Y = N1 * N2
320  PRINT "WHAT IS THE ANSWER TO ";N1;" * ";N2
325  INPUT S
330  IF S = Y THEN  PRINT "GOOD WORK !!":C = C + 1: GOTO 345
335  PRINT "SORRY, BUT THE CORRECT ANSWER IS ";Y
345  NEXT X
350  GOTO 600
400  REM        MIXTURE
401  FOR X = 1 TO 20
402  LET W =  INT ( RND (1) * 3) + 1
403  LET N1 =  INT ( RND (1) * 11)
404  LET N2 =  INT ( RND (1) * 10)
405  IF W = 1 THEN  GOTO 420
406  IF W = 2 THEN  GOTO 440
407  GOTO 460
420  REM  ADDING
421  LET Y = N1 + N2
422  PRINT "WHAT IS THE ANSWER TO ";N1;" + ";N2
425  INPUT S
430  IF S = Y THEN  PRINT "GOOD WORK !!":C = C + 1: GOTO 480
435  PRINT "SORRY, BUT CORRECT ANSWER IS ";Y: GOTO 480
440  REM   SUBTRACTING
441  IF N1 ) N2 THEN  GOTO 445
442  LET E = N1:N1 = N2:N2 = E
445  LET Y = N1 - N2
446  PRINT "WHAT IS THE ANSWER TO ";N1;" - ";N2
447  INPUT S
448  IF S = Y THEN  PRINT "GOOD WORK !!":C = C + 1: GOTO 480
450  PRINT "SORRY, BUT THE CORRECT ANSWER IS ";Y: GOTO 480
460  REM   MULTIPLICATION
461  LET Y = N1 * N2
462  PRINT "WHAT IS THE ANSWER TO ";N1;" * ";N2
463  INPUT S
465  IF S = Y THEN  PRINT "GOOD WORK !!":C = C + 1: GOTO 480
466  PRINT "SORRY, BUT THE CORRECT ANSWER IS ";Y
480  NEXT X
600  REM   COMPUTING AVERAGE CORRECT
605  PRINT "YOU HAD ";C;" CORRECT OUT OF 20."
610  PRINT : PRINT "WHICH IS AN AVERAGE OF ";(C / 20) * 100;" %"
620  IF C / 20 < .7 THEN  PRINT "YOU MIGHT WANT TO TRY THIS SECTION AGAIN
     ": GOTO 999
630  PRINT "SO, YOU DID QUITE WELL !!"
999  END
```

General Topic : Arithmetic and Measurement

Specific Topic : Whole Number Operations: Drill and Practice with Addition, Subtraction or Multipication

Objectives : Students will be able to practice selected arithmetic facts.

Description : Student selects either addition, subtraction or multiplication. The computer generates random numbers for 20 problems of the chosen type. As the student enters the answer, the computer responds with "Correct" or "Wrong, the answer is ____." At the end, the number and percent correct is displayed.

Uses and Extensions : The program may be modified to have the computer generate random types of problems, such as a combination of different types of problems. See the activity "Whole Number Operations" under Computation for this extension.

]LIST

```
5  HOME : VTAB 15
10  PRINT "WHICH OPERATION DO YOU WANT TO PRACTICE "
30  PRINT : PRINT : PRINT : PRINT "1)  ADDITION"
40  PRINT "2)  SUBTRACTION"
50  PRINT "3)  MULTIPLICATION"
60  PRINT : PRINT
70  INPUT "TYPE  1,2, OR 3 ";N
100  HOME :C = 0
1010  FOR X = 1 TO 20
1011  HOME
1020 R1 =  INT ( RND (1) * 200) + 1
1025 R4 =  INT ( RND (1) * 5000) + 1
1026 R5 =  INT ( RND (1) * 975) + 1
1027 R6 =  INT ( RND (1) * 40) + 1
1028 R2 =  INT ( RND (1) * 100) + 1
1032 R4 = R4 + 876
1035  ON N GOTO 2000,3000,3500
1037  GOTO 5
1040  ON N GOTO 2000,3000,3500
1041  PRINT : VTAB 8
1070  INPUT "   TYPE THE ANSWER HERE ---) ";A
1080  IF A = R3 THEN 1500
1090  PRINT : PRINT "WRONG ANSWER "
1100  PRINT : PRINT : PRINT "THE ANSWER IS ";R3: FOR S = 1 TO 750: NEXT S

1110  NEXT X: GOTO 4000
1500 C = C + 1
1510  PRINT : PRINT : PRINT "CORRECT !": FOR S = 1 TO 750: NEXT S
1610  NEXT X: GOTO 4000
2000 R3 = R1 + R2: PRINT  TAB( 8);R1;" + ";R2;" = ": GOTO 1041
3000 R3 = R4 - R5: PRINT  TAB( 8);R4;" - ";R5;" = ": GOTO 1041
3500 R3 = R1 * R6: PRINT  TAB( 8);R1;" * ";R6;" = ": GOTO 1041
4000  PRINT "END": PRINT : PRINT : PRINT
4020  PRINT "YOUR SCORE IS: ";C;" QUESTIONS CORRECT OUT OF 20 PROBLEMS"
4030  PRINT : PRINT : PRINT "THAT IS ";C * 5;"%"
4040  END
```

General Topic : Arithmetic and Measurement

Specific Topic : Areas and perimeters of squares and rectangles

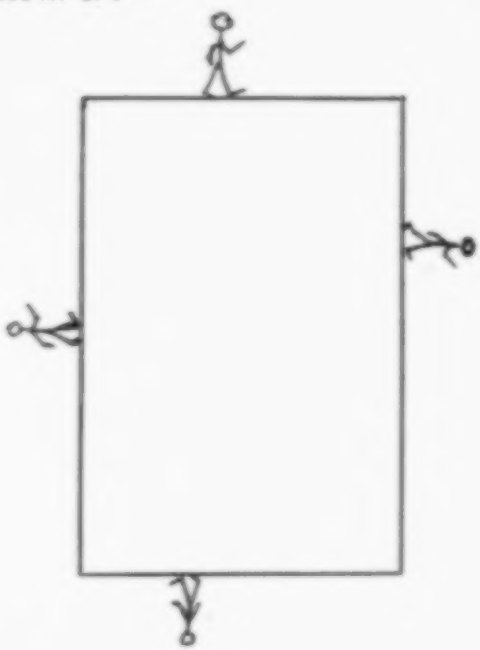
Objective : a). Given the width and length of a rectangle, the student will determine the area and perimeter, and be able to check his or her answer with that of the computer. b). For more advanced students, the student will be able to determine the probability of a square being randomly generated.

Description : The activity is based on a computer program that randomizes the width and length for a rectangle. The student determines and enters the area and perimeter. The computer will indicate whether or not the student's answer is correct or incorrect. For the probability experiment, the student determines the probability of rectangles being squares by keeping a table of results.

Uses and Extensions : Drill and practice to reinforce the relationship of length and width to that of area and perimeter. The program may be modified to determine the probability of a rectangle being a square. For the probability extension, the student determines the probability of rectangles being squares by keeping a table of results.

]LIST

```
10  PRINT "A VARIETY OF LENGTHS AND WIDTHS DIMENSIONS "
20  PRINT "FOR RECTANGLES WILL BE GIVEN. THE "
30  PRINT "OBJECT IS TO CORRECTLY COMPUTE THE AREA "
35  PRINT "AND PERIMETER FOR EACH."
40  PRINT : PRINT : PRINT
50 A =  INT (37 *  RND (1)) + 1
60  LET B =  INT (4 *  RND (1)) + 1
70  PRINT "WHAT IS THE AREA AND THE PERIMETER"
80  PRINT "FOR A RECTANGLE WITH THE LENGTH AND WIDTH OF: "
100  PRINT : PRINT A;"     AND   ";B
110  LET AREA = A * B
120  LET PERI = 2 * (A + B)
130  PRINT : INPUT "WHAT IS THE AREA ";E
140  IF AREA = E THEN 160
150  PRINT "WRONG. THE AREA IS ";AREA
151  GOTO 170
160  PRINT "CORRECT!!"
170  PRINT : INPUT "WHAT IS THE PERIMETER ";P
180  IF PERI = P THEN 200
190  PRINT "WRONG. THE PERIMETER IS ";PERI
191  GOTO 210
200  PRINT "CORRECT"
210  GOTO 40
220  END
```



General Topic : Arithmetic and Measurement

Specific Topic : Interior Decorating Measurements

Objective : To teach students the relationship between the size (dimensions) of a room and the amount of paint needed (surface area) to paint the room.

Description : The activity requires that students measure a room and acquire paint coverage figures (square feet per gallon). The program aids in calculating the total quantity of paint required.

Uses and Extensions : Use the activity to encourage measurement and calculation exercises or to fill in work sheets.

The activity can be extended by considering the number of doors and windows involved, by using different coverages for wall paint versus ceiling paint, by introducing costs, or by adding additional rooms.

```
]LIST

100  PRINT "YOU ARE PAINTING YOUR ROOM AND NEED TO FIND OUT IF YOU HAVE E
     NOUGH PAINT"
110  PRINT : PRINT "INPUT THE FOLLOWING": PRINT
120  INPUT "LENGTH OF ROOM ";L
125  INPUT "WIDTH OF ROOM ";W
130  INPUT "SQUARE FEET PER GALLON OF PAINT ";S
140  INPUT "GALLONS OF PAINT YOU HAVE NOW ";B
150  LET H = B: PRINT : LET A = 2 * L * H + 2 * W * H
160  LET G = A / S: IF B <  = G THEN  GOTO 180
170  PRINT "YOU ARE OKAY, YOU ONLY NEED ";G;" GALLONS.": GOTO 185
180  PRINT "BUY MORE PAINT. YOU NEED ";G - B;" GALLONS MORE."
185  END
```

```
]RUN
YOU ARE PAINTING YOUR ROOM AND NEED TO FIND OUT IF YOU HAVE ENOUGH PAINT

INPUT THE FOLLOWING

LENGTH OF ROOM 9
WIDTH OF ROOM 13
SQUARE FEET PER GALLON OF PAINT 20
GALLONS OF PAINT YOU HAVE NOW 9

BUY MORE PAINT. YOU NEED 8.6 GALLONS MORE.
```

General Topic : Arithmetic and Measurement

Specific Topic : Percentages

Objective : Student will gain practice in calculating percentages.

Description : The activity allows students to use a computer to calculate percentages. If the answer is an integer, a pictorial display will be given along with the answer. Many examples are given, after which the program can be changed to allow entering any number and percent.

Uses and Extensions : Use the program as it is, recording the results in a chart similar to the one below:

Problem	Percent (P)	Number (N)	Answer (A)
1)			
2)			
3)			
⋮	⋮	⋮	⋮

Alter the program by typing these lines:

```
50 INPUT "ENTER THE PERCENT TO USE ";P
60 INPUT "ENTER THE NUMBER ";N
```

Enter your own numbers and percents. Record the results in the chart.

What do you notice about the size of the percents in problem numbers 6,8,13,14? What do you notice about the answers in problems 6,8,13,14?

LIST

```
10  HOME : PRINT "THIS PROGRAM WILL DETERMINE THE PERCENT OF A NUMBER AND
      DISPLAY THE PERCENT, "
20  PRINT "NUMBER AND ANSWER"
60  READ P,N: IF P = 0 THEN  END
70 A = N * P / 100
80  PRINT "PERCENT","NUMBER","ANSWER"
81  PRINT P,N,A: IF A =  INT (A) THEN 200
85  PRINT : PRINT : PRINT
90  GOTO 60
200  PRINT : PRINT
210  PRINT "A DISPLAY FOR THE NUMBER (N) AND THE PERCENTAGE (A) ANSWER."
220  PRINT : PRINT : PRINT
230  FOR R = 1 TO N: PRINT "N";: NEXT R
260  PRINT : PRINT
270  FOR S = 1 TO A: PRINT "A";: NEXT S
300  PRINT : PRINT : PRINT
305  FOR X = 1 TO 300: NEXT X: RESTORE : GOTO 60
315  DATA  7,200,40,80,16,300,2,150,10,70,22,400,20,90,9,400,4,350,5,60,.
     5,200,.8,500,2.5,400,.5,400,.5,800,0,0
```

General Topic : Computation and Problem Solving

Specific Topic : Understanding Computational Errors

Objectives : Students will be able:

(1) to discover discrepancies that occur when using a computer to solve problems and make calculations.

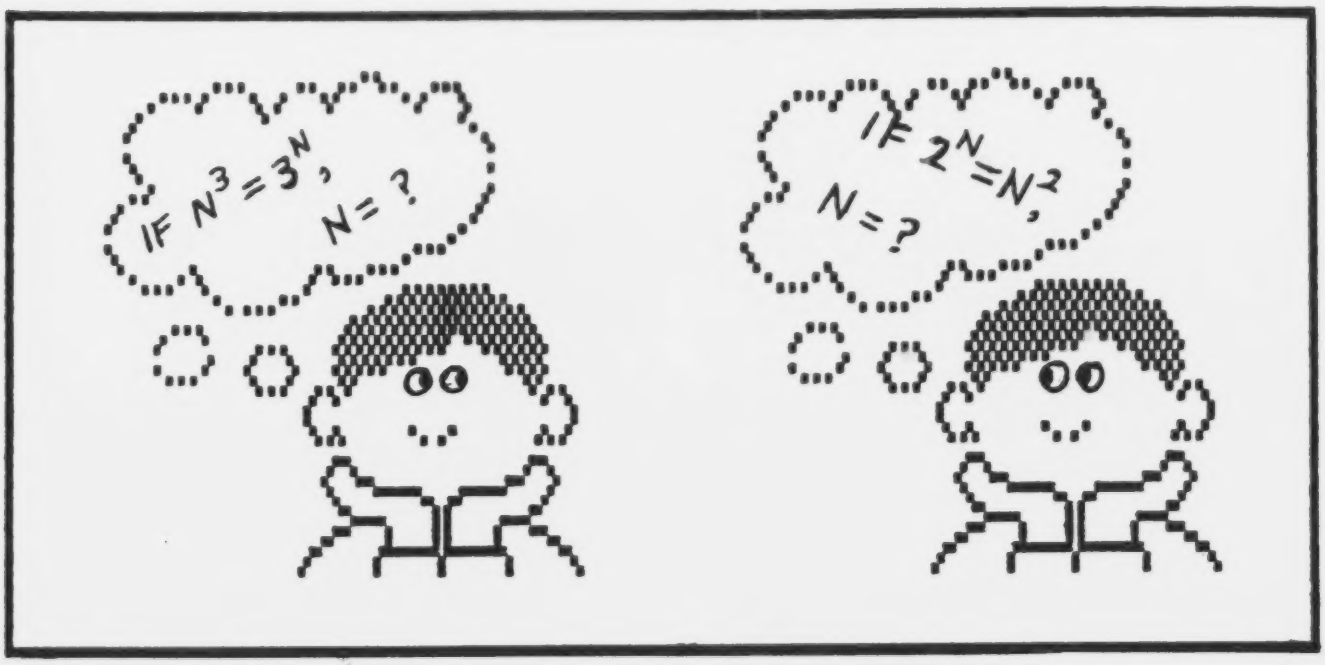
(2) to find the square and cube of numbers using two approaches.

(3) to use two methods of finding the square root of a number.

(4) to conclude how important it is to understand mathematical concepts when using a microcomputer to solve problems.

Description : Students will enter the programs on the work sheets and will be able to answer the questions.

Uses and Extensions : The programs in this activity can be run on any microcomputer. If more than one type of microcomputer is available, it is advantageous to run the programs on all the machines. It is surprising to note that different micros produce different results. Also, it is difficult to include answers to all activities in this segment since different computers produce different results. For the Apple computer, replace the arrow with a ^ in the programs in this activity. The teacher will be requested to instruct the students on how to obtain hard copies of the program runs.



1. Enter the following in the immediate mode and record your results.

(a) PRINT 7↑2 _____

(b) PRINT 30↑2 _____

(c) PRINT 31↑2 _____

(d) PRINT 30*30 _____

(e) PRINT 31*31 _____

(f) PRINT 9↑3 _____

(g) PRINT 9*9*9 _____

2. If a different microcomputer is available, try the above on another machine. Are the results the same ? _____

3. Enter the following BASIC program:

```
10 N=1
15 PRINT N,N*N,N↑2
20 N=N+1
25 IF N>100 THEN 35
30 GOTO 15
35 END
```

4. RUN the program above. Do you notice any peculiarities as the results appear on the screen ? _____

5. Ask your teacher how to make this program run on the printer. Run the program on the line printer to obtain a print out (hard copy) of the results. Use your hard copy to take a closer look at the results and answer the following questions:

(a) How many discrepancies occurred ? _____

(b) List the numbers for which there were discrepancies in the above cases.

_____

(c) What is the value of the discrepancy in the above cases ?

_____

(d) Is the value of the discrepancy always the same ? _____

(e) What do you think would happen if this program were run for different values of N between 100 and 200 inclusive ? _____

6. Make the following changes in the program:

```
10 N = 100
25 IF N > 200 THEN 35
```

7. RUN the program now. Record the discrepancies.

_____

Were they what you expected to get ? _____

8. Is there a pattern of errors that occur ? _____ To answer this question, make the following changes in the program:

```
10 N = 1
25 IF N > 500 THEN 35
```

9.RUN this program and note the errors. Look for a pattern to the errors. Is there one ?? _____

10. Hopefully this has aroused your curiosity. Let's see what would happen if we were to cube the integers from 1 to 500, inclusive.

(a) Do you expect to get the same errors ? _____

(b) Will there be more errors ? _____

(c) Will the discrepancy factor be the same ? _____

(d) Enter the following BASIC program:

```
 5 N=1
10 PRINT N,N*N*N, N↑3
15 N=N+1
20 IF N>500 THEN 30
25 GOTO 10
30 END
```

(e) What happened ? _____ How many mistakes were there ? _____ Is this what you had expected to occur ? _____

General Topic : Computation and Problem Solving

Specific Topic : Vacation Cost Problems

Objective : Students will be able to calculate travel costs using different mileages, gas prices and time values.

Description : The activity requires that students enter and use a program in order to find the answer to various vacation cost problems.

Uses and Extensions : The activity and accompanying program can be used to simulate planning process, complete with data entry, calculations and analysis.

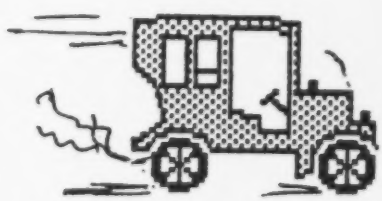
An activity sheet that includes questions like the following can be quite useful. "You are planning on taking a driving vacation with your parents. Your father wants to know how long it will take you to get to your destination and how much money gas will cost."

Extensions include adding food and lodging costs to the activity or changing the mode of travel to plane, boat or train.

```
]LIST

100  PRINT "TAKING A VACATION ? USE THIS PROGRAM TO CALCULATE YOUR DRIVIN
     G TIME AND ";
110  PRINT "COST OF GASOLINE": PRINT
140  INPUT "MILES TO TRAVEL ";M
160  INPUT "MILES PER GALLON ";A
180  INPUT "PRICE PER GALLON ";P
200  INPUT "MILES PER HOUR ";S
220  LET T = M / S
230  LET C = (M / A) * P
240  PRINT "IT WILL TAKE ";T;" HOURS AND COST ";C;" DOLLARS."
250  END
```

```
]RUN
TAKING A VACATION ?

MILES TO TRAVEL 1200
MILES PER GALLON 22
PRICE PER GALLON 1.34
MILES PER HOUR 55
IT WILL TAKE 21.8181818 HOURS AND COST 73.0909091 DOLLARS.
```

```
]RUN
TAKING A VACATION ?

MILES TO TRAVEL 50
MILES PER GALLON 30
PRICE PER GALLON .99
MILES PER HOUR 30
IT WILL TAKE 1.66666667 HOURS AND COST 1.65 DOLLARS.
```



General Topic : Computation and Problem Solving

Specific Topic : Compound Interest Formula

Objective : Students will be able to solve problems involving compound interest.

Description : This activity uses a program that requires students to enter a principal, length of time held in months, and a rate of interest in decimal form. The computer will produce the interest (profit) earned, rounded to the nearest cent, and display it along with the data entered.

Uses and Extensions : Students enter the program and data, keeping track of their results and answer the following questions.

1. What interest rate would you need to yield $253 on a principal of $1000 held for 30 months ?

2. By repeated trials, what principal would be needed to produce $625 in 18 months at 9% interest ?

3. What other unknown quantity can you estimate ?

```
]LIST

10  HOME : INPUT "ENTER THE PRINCIPAL ";P
20  INPUT "ENTER THE NUMBER OF MONTHS THE PRINCIPAL IS HELD ";N
30  INPUT "ENTER THE INTEREST RATE IN DECIMAL FORM ";I
35 T =  INT (P * ((1 + I / 12) ^ N - 1) * 100 + .5) / 100
40  PRINT "$";P;" INVESTED AT ";10 * I;"% FOR ";N;" MONTHS YIELDS $";T;"
     PROFIT."
60  INPUT "HIT RETURN TO CONTINUE ";A$
70  PRINT : PRINT
75  GOTO 10
80  END
```

```
]RUN
ENTER THE PRINCIPAL 400
ENTER THE NUMBER OF MONTHS THE PRINCIPAL IS HELD 40
ENTER THE INTEREST RATE IN DECIMAL FORM .25
$400 INVESTED AT 2.5% FOR 40 MONTHS YIELDS $512.54 PROFIT.
```

General Topic : Computation and Problem Solving

Specific Topic : Number Patterns

Objective : Students will be able to determine a number pattern, given the first twelve terms. After determining the pattern, students will be able to produce the next term.

Description : One student enters the program and runs it. This student asks a friend to figure out the pattern. That is, the friend is to state a rule or formula for generating the terms in the pattern. They may take turns creating and entering patterns by changing line 40 of the program. They may then invent their own patterns to challenge each other.

Uses and Extensions : Use as indicated in the Description, and then try these variations by entering any one of the following for line 40:

```
40 R = 2 ^ N          40 R = N * (N + 1)

40 R = N * N * N      40 R = 5 * N - 3

40 R = 2 * N + 1      40 R = N ^ N
```

Make up your own expession and try it. Note that lines 20 and 25 can be omitted.

For the Fibonacci sequence, enter the following:

```
 8 DIM R(12)
30 R(1)=1: R(2)=1
35 FOR N= 3 TO 12
40 R(N)=R(N-1) + R(N-2)
45 NEXT N
50 FOR N=1 TO 12
55 PRINT R(N)
60 NEXT N
```

You may also amend the program to allow for entering and checking the next term.

]LIST

```
5  HOME
20  PRINT "ASK A FRIEND TO IDENTIFY THIS NUMBER PATTERN"
25  PRINT "THE NUMBER PATTERN"
27  PRINT : PRINT
30  FOR N = 1 TO 12
40  LET R = N * N
50  PRINT N,R
60  NEXT N
70  PRINT : PRINT : PRINT
```

General Topic : Computation and Problem Solving

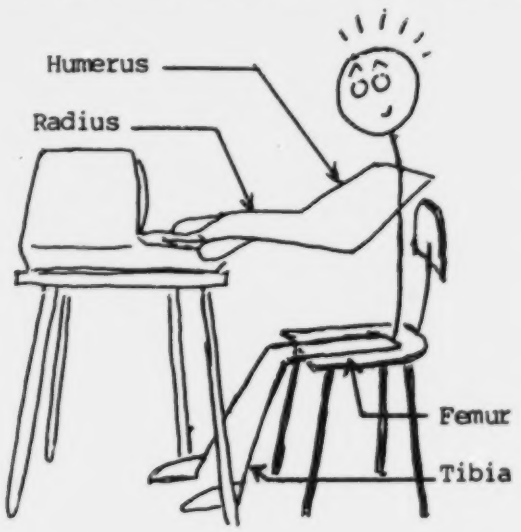
Specific Topic : Bio-Mathematics Application

Objective : Students will be able to utilize a program to find the height of an individual given one of four bone lengths in centimeters. Students will be able to determine the size relations of the four bones in a given individual.

Description : This activity is adapted from the article "Application: Mathematics in Forensic Science"; Mathematics Teacher, George Knill, Hamilton, Ontario, Feb. 1981. The program used is based on forensic formulas and has been written to allow the computer to find the height of the individual after the user supplies the requested information.

Uses and Extensions : Students should be instructed to type in the program given. They will be asked to enter the sex, bone, bone size in centimeters, and age of the person. The computer will print out the approximate height of the individual in centimeters.

A set of four questions have been included to accompany the program. However, there are a number of other questions possible, depending on the background of the students.



Suggested Questions for use in Hand-outs.

1. The tibia of a 20 year old male measures 36.9 cm. What is his height?

2. The radius of a 33 year old female measures 21 cm. What is her height?

3. Measure your humerus bone. Compute your height. Was the computer's answer close to your answer?

4. CHALLENGE: Rewrite the program to compute the length of the bones given the person's sex, height in cm and age.

```
]LIST

5  HOME
10  INPUT "SEX; M=MALE  F=FEMALE ";S$
20  INPUT "BONE: F=FEMUR  T=TIBIA  H=HUMERUS  R=RADIUS ";B$
25  INPUT "LENGTH OF BONE (IN CENTIMETERS) ";B
30  INPUT "AGE ";A
40  IF S$ = "M" AND B$ = "F" THEN  LET H = 69.089 + 2.238 * B
50  IF S$ = "M" AND B$ = "T" THEN  LET H = 81.688 + 2.392 * B
60  IF S$ = "M" AND B$ = "H" THEN  LET H = 73.57 + 2.97 * B
70  IF S$ = "M" AND B$ = "R" THEN  LET H = 80.405 + 3.65 * B
80  IF S$ = "F" AND B$ = "F" THEN  LET H = 61.412 + 2.317 * B
90  IF S$ = "F" AND B$ = "T" THEN  LET H = 72.572 + 2.533 * B
100  IF S$ = "F" AND B$ = "H" THEN  LET H = 64.977 + 3.144 * B
110  IF S$ = "F" AND B$ = "R" THEN  LET H = 73.502 + 3.876 * B
120  IF A > 30 THEN  LET H = H - (.06 * (A - 30))
130  PRINT "THE APPROXIMATE HEIGHT OF THE PERSON IN CM IS ";H
135  PRINT : PRINT
136  PRINT "THE APPROXIMATE HEIGHT OF THE PERSON IN INCHES IS ";H / 2.5
140  END
```

```
]RUN
SEX; M=MALE  F=FEMALE (M)
BONE: F=FEMUR  T=TIBIA  H=HUMERUS  R=RADIUS (T)
LENGTH OF BONE (IN CENTIMETERS) (30)
AGE (21)
THE APPROXIMATE HEIGHT OF THE PERSON IN CM IS 153.448


THE APPROXIMATE HEIGHT OF THE PERSON IN INCHES IS 61.3792



]RUN
SEX; M=MALE  F=FEMALE (F)
BONE: F=FEMUR  T=TIBIA  H=HUMERUS  R=RADIUS (F)
LENGTH OF BONE (IN CENTIMETERS) (40)
AGE (22)
THE APPROXIMATE HEIGHT OF THE PERSON IN CM IS 154.092


THE APPROXIMATE HEIGHT OF THE PERSON IN INCHES IS 61.6368
```

General Topic : Algebra

Specific Topic : Addition, Subtraction, or Multiplication of Signed Numbers

Objective : Students will be able to practice basic operations with signed numbers.

Description : This activity uses a program that randomly selects one of the above operations and the numbers to be used. If the student's answer is correct, a bell will ring (assuming you are using an Apple computer and CHR$(7)) and a new operation is randomly selected. If the answer is wrong, no bell rings and another problem of the same type is presented. When ten out of ten problems are worked correctly, twenty bells ring !

Uses and Extensions : Use for drill and practice, as indicated in the program. Extensions may be based on teacher generated work sheets.

]LIST

```
10  REM  ONE BELL FOR EACH CORRECT ANSWER
15  REM  20 BELLS FOR TEN CORRECT ANSWERS
20  PRINT "THIS IS A PROGRAM TO PRACTICE OPERATIONS WITH SIGNED NUMBERS !
    !":K = 0
40 A =  INT ( RND (1) * 3) + 1
50 B =  INT ( RND (1) * 20) + 1
60 C =  INT ( RND (1) * 10) + 1
70 D =  INT ( RND (1) * 10) + 1
80 E =  INT ( RND (1) * 10) + 1
90 F = ( - 1) ^ D:G = ( - 1) ^ E:H = F * B:I = G * C
130  PRINT : PRINT : PRINT
140  ON A GOTO 170,240,290
170  PRINT "(";H;") + (";I;") = ";
180  INPUT J
200  IF J = H + I THEN  PRINT "CORRECT!!": PRINT  CHR$ (7): FOR CX = 1 TO
     750: NEXT CX: HOME :K = K + 1: IF K = 10 THEN  GOTO 340
210  IF J = H + I THEN  GOTO 40
230  PRINT "SORRY, TRY AGAIN !": PRINT : GOTO 170
240  PRINT "(";H;") - (";I;") = ";
250  INPUT J
260  IF J = H - I THEN  PRINT "RIGHT YOU ARE !!": PRINT  CHR$ (7): FOR CX
      = 1 TO 750: NEXT CX: HOME :K = K + 1: IF K = 10 THEN 340
270  IF J = H - I THEN  GOTO 40
280  PRINT "TOO BAD, TRY AGAIN !": PRINT : GOTO 240
290  PRINT "(";H;") X (";I;") = ";
300  INPUT J
310  IF J = H * I THEN  PRINT "NICE JOB, YOUR CORRECT!!": PRINT  CHR$ (7)
     : FOR CX = 1 TO 750: NEXT CX: HOME :K = K + 1: IF K = 10 THEN 340
320  IF J = H * I THEN  GOTO 40
330  PRINT "NO, THAT'S NOT CORRECT !! TRY AGAIN!!": PRINT : GOTO 290
340  FOR DX = 1 TO 1000: NEXT DX
380  FOR BL = 1 TO 20: PRINT  CHR$ (7): NEXT BL
```

General Topic : Algebra

Specific Topic : Sequences

Objective : Students will be able to gain experience in determining the next term of a geometric or arithmetic progression.

Description : The activity uses a program that prints a sequence of whole numbers. Students figure out, guess, or otherwise attempt to determine what the next number should be. They then enter their answer to see if they are correct.

Uses and Extensions : The activity can be used to demonstrate geometric and arithmetic sequences, to demonstrate infinite series and to build problem solving skills relative to finding generating terms.

Extensions include adding a counter to keep track of the number correct and changing the generating lines to produce different sequences, including some that are neither arithmetic nor geometric.

```
]LIST

10  HOME
100  PRINT "THIS PROGRAM PRINTS A SEQUENCE OF NUMBERS "
105  PRINT "SEE IF YOU CAN FIGURE OUT WHAT THE NEXT NUMBER SHOULD BE "
110 S =  INT ( RND (1) * 2) + 1
115 A =  INT ( RND (1) * 5) + 1
120 B =  INT ( RND (1) * 10) + 1
125 C =  INT ( RND (1) * 3) + 1
130  IF C = 1 THEN 125
140  IF S = 1 THEN 165
145  REM  THIS IS A GEOMETRIC SEQUENCE
150 N = A * C ^ I
155  GOTO 170
160  REM  THIS IS AN ARITHMETIC SEQUENCE
165 N = A + I * B
170 I = I + 1
175  IF I > 5 THEN 190
177 N =  INT (N): PRINT N;"    ";
185  GOTO 140
190  INPUT M: IF M = N THEN 210
200  PRINT "SORRY, THE NEXT NUMBER IS ";N: GOTO 220
210  PRINT "YOU WERE RIGHT, CONGRATULATIONS !!"
220  END
```

```
THIS PROGRAM PRINTS A SEQUENCE OF NUMBERS
SEE IF YOU CAN FIGURE OUT WHAT THE NEXT NUMBER SHOULD BE
4    10    16    22    28    ?34
YOU WERE RIGHT, CONGRATULATIONS !!


]RUN
THIS PROGRAM PRINTS A SEQUENCE OF NUMBERS
SEE IF YOU CAN FIGURE OUT WHAT THE NEXT NUMBER SHOULD BE
3    9    27    81    243    ?621
SORRY, THE NEXT NUMBER IS 729
```

General Topic : Algebra

Specific Topic : Pascal's Triangle

Objective : Students will be able to determine the underlying relationship found in Pascal's Triangle.

Description : The activity uses a program to generate Pascal's triangle, and then has students answer questions on a worksheet.

Uses and Extensions : After running the program, students may use the program listing or an activity sheet, to discover the fact that each new line of the triangle can be obtained fron the preceding line.

Extensions include using the triangle in exercises that call for binomial expansions or for calculating the Fibonacci numbers.

```
]LIST

100  PRINT "HERE IS PASCAL'S TRIANGLE"
110  DIM E(10,10)
120  LET M = 35
130  REM  PUT NUMBERS INTO ARRAY
140  LET E(1,1) = 0
150 N = 8
160  FOR R = 2 TO N
170 E(R,1) = 1
180  FOR S = 2 TO R - 1
190 E(R,S) = E(R - 1,S - 1) + E(R - 1,S)
200  NEXT S
205 E(R,R) = 1
210  NEXT R
220  REM  NOW PRINT TRIANGLE
240  FOR I = 1 TO N
250  PRINT  TAB( M - (I - 1) * 3);
260  FOR J = 1 TO I
270  PRINT E(I,J);"     ";
280  NEXT J
290  PRINT
300  NEXT I
310  END
```

```
]RUN
HERE IS PASCAL'S TRIANGLE!
                    0
                1       1
            1       2       1
        1       3       3       1
    1       4       6       4       1
1       5       10      10      5       1
    6       15      20      15      6       1
7       21      35      35      21      7       1
```

General Topic : Algebra

Specific Topic : The Quadratic Formula

Objective : The student will be able to use the computer to generate all of the solutions of a quadratic equation of the form AX^2 + BX + C = 0.

Description : This activity will determine all the roots of a quadratic equation.

Uses and Extensions : This activity can be used for graphing, algebra and probability lessons. As a graphing extension, add FOR/NEXT loops to generate A,B and C within the program. Next add counters at lines 90 and 100 for the number of real, unique and double roots produced.

```
]LIST

10  PRINT "THIS PROGRAM WILL DETERMINE THE SOLUTIONS TO"
20  PRINT "AN EQUATION OF THE FORM AX^2 + BX + C"
30  PRINT : PRINT
40  INPUT "ENTER A VALUE FOR A ";A
45  IF A = 0 THEN  GOTO 40
50  INPUT "ENTER A VALUE FOR B ";B
60  INPUT "ENTER A VALUE FOR C ";C
70  PRINT : PRINT
75 REAL =  - B / (2 * A)
80 D = B ^ 2 - 4 * A * C
90  IF D = 0 THEN  PRINT REAL;" ONE REAL ROOT": END
100  IF D > 0 THEN  PRINT REAL +  SQR (D) / (2 * A);" AND ";REAL -  SQR (
     D) / (2 * A);" ARE THE TWO REAL ROOTS": END
110 IMAG =  SQR ( - D) / (2 * A)
120  PRINT REAL;"+";IMAG;"*i"
130  PRINT REAL; - IMAG;"*i"
140  END
```

```
]RUN
THIS PROGRAM WILL DETERMINE THE SOLUTIONS TO
AN EQUATION OF THE FORM AX^2 + BX + C

ENTER A VALUE FOR A 1
ENTER A VALUE FOR B 2
ENTER A VALUE FOR C 1

-1, ONE REAL ROOT
```

```
ENTER A VALUE FOR A 2
ENTER A VALUE FOR B 1
ENTER A VALUE FOR C 2

-.25+.968245837*i
-.25-.968245837*i
```

General Topic : Geometry

Specific Topic : The Pythagorean Theorem

Objective : Students will be able to apply the Pythagorean Theorem, its converse, and related theorems regarding obtuse and acute triangles.

Description : The activity allows students to use a computer program that accepts the lengths of the three sides of a triangle, and then determines and displays information as to whether the triangle formed is acute, right, or obtuse. The program also prints out information when no triangle is formed.

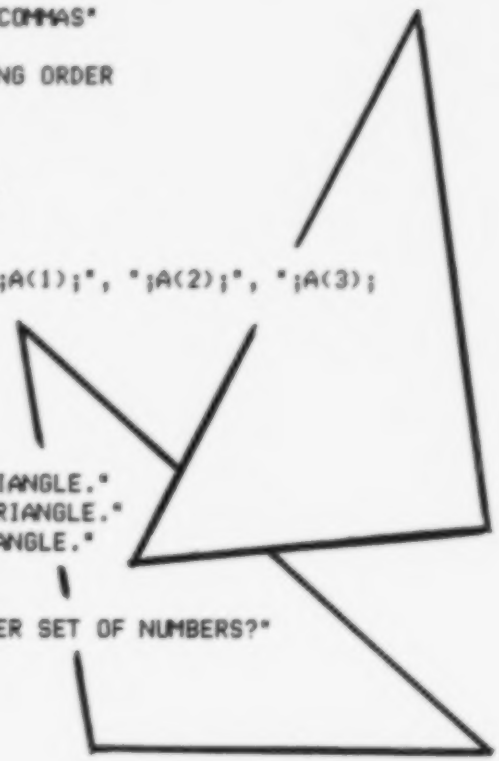
Uses and Extensions : Please be aware that difficulties arise under certain conditions when using exponential notation such as A(1) ^ A(2) = A(3)^2. Due to the fact that the computer uses logs to compute exponents, decimals may become involved when squaring an integer. Therefore, it was necessary to use A(1) * A(1) instead of A(1)^2 (see lines 500 and 510 of the program).

Since the program is lengthy, you may find it easier to store the program on a disk.

An interesting extension is to have those 10th and 11th grade geometry students with programming experience write and use their own versions of an interactive program that uses the Pythagorean Theorem and its related theorems to determine if a given triangle is obtuse, acute or right.

LLIST

```
150  PRINT "TYPE YOUR THREE SIDES SEPARATED BY COMMAS"
170  INPUT A(1),A(2),A(3)
185  REM  BUBBLE SORT: ARRANGE SIDES IN ASCENDING ORDER
230  FOR N = 1 TO 2
240  FOR I = 1 TO 2
250  IF A(I) < A(I + 1) THEN 290
260 TEMP = A(I):A(I) = A(I + 1):A(I + 1) = TEMP
290  NEXT I
300  NEXT N
400  PRINT "IF THE SIDES OF YOUR TRIANGLE ARE ";A(1);", ";A(2);", ";A(3);
     " THEN ";
410  IF A(1) + A(2) <  = A(3) THEN 550
420  IF A(2) + A(3) <  = A(1) THEN 550
430  IF A(1) + A(3) <  = A(2) THEN 550
500  LET L = A(1) * A(1) + A(2) * A(2)
510  LET M = A(3) * A(3)
520  IF L > M THEN  PRINT "YOU HAVE AN ACUTE TRIANGLE."
530  IF L < M THEN  PRINT "YOU HAVE AN OBTUSE TRIANGLE."
540  IF L = M THEN  PRINT "YOU HAVE A RIGHT TRIANGLE."
545  GOTO 600
550  PRINT "NO TRIANGLE IS FORMED."
600  PRINT : PRINT "WOULD YOU LIKE TO TRY ANOTHER SET OF NUMBERS?"
610  INPUT "TYPE YES OR NO. ";A$
620  IF A$ = "YES" THEN 150
630  END
```



SAMPLE WORKSHEET

1. Enter the attached BASIC program (or load it from a disk).

2. Enter RUN, and respond to the questions asked. That is, you will have to enter three sides (lengths) of a triangle.

3. Can you decide whether the triangle formed with the given sides is acute, obtuse or right? Consider the possibility that NO TRIANGLE will be formed. See if you can guess the correct answer before entering your data.

Examples:

a) 12,5,13
b) 8,18,15
c) 9,12,14
d) 23,9,13
e) 0.7,0.7,1
f) 1.1,6.0,6.1
g) 0.6,0.8,1
h) 2*(SQR(15)),6,5
i) SQR(7),SQR(15),SQR(8)
j)2,4,3
k) 7,6,5

4. State the Pythagorean Theorem.

5. Under what conditions will your triangle be a right angle?

6. Can you discover when a triangle will be obtuse? Acute?

7. Why are there situations when no triangle is formed?

General Topic : Number Theory

Specific Topic : The Division Algorithm

Objective : Students will be able to investigate digit patterns in division problems.

Description : This program uses the division algorithm to perform long division to any desired number of places. The resulting data is applicable in number theory and statistics for such investigations as digit patterns in quotients, cycles of repetition and distribution of digits. An interesting sequence of digits for students to examine is the result of 355/113 as an approximation for pi.

Uses and Extensions : Use to emphasize place values, to search for number patterns, to generate data for statistics, or to approximate mathematical constants such as pi and e.

Note that the program can be used for the study of error generation, particularly when dividing by a decimal.

```
]RUN
NUMERATOR =355
DENOMINATOR =113
NUMBER OF DIGITS IN QUOTIENT =10

3.141592920


]RUN
NUMERATOR =1237
DENOMINATOR =524
NUMBER OF DIGITS IN QUOTIENT =22

2.360687022900763358778
```

```
]LIST

 5  INPUT "NUMERATOR =";N
 10  INPUT "DENOMINATOR =";D
 15  INPUT "NUMBER OF DIGITS IN QUOTIENT =";N1
 17  PRINT
 20  FOR D1 = 1 TO N1
 30 Q =  INT (N / D)
 40  PRINT Q;
 45  IF D1 = 1 THEN  PRINT ".";
 50 N = 10 * (N - Q * D)
 60  NEXT D1
 70  END
```

```
]RUN
NUMERATOR =58763
DENOMINATOR =675.2
NUMBER OF DIGITS IN QUOTIENT =50

87.030509472526751954936539615157765757059537923233 7

]RUN
NUMERATOR =3124
DENOMINATOR =513
NUMBER OF DIGITS IN QUOTIENT =79

6.089668615984405458089668615984405458089668615984405458089668615984405458089
```

General Topic : Number Theory

Specific Topic : Clock and Modular Arithmetic

Objective : Students will explore clock or modular arithmetic using the following program.

Description : The activity uses a program to change a base 10 number into a base K number.

Uses and Extensions : (a). Refer to the student worksheet on the following page. When running, enter CONTROL-C or Break, depending upon the machine, to end program.

(b). Line 5 of the program may be re-entered as 5 INPUT "ENTER K ";K to allow for entering arbitrary non-zero integer bases. Doing this provides the answer to question 8 on the student activity sheet.

(c). The activity can be extended to allow for the adding and subtracting of modular numbers. This is recommended for more advanced students.

]LIST

```
5 K = 12
10  PRINT "THIS PROGRAM WILL CONVERT A DECIMAL"
20  PRINT "NUMBER TO A 12-HOUR CLOCK NUMBER."
30  PRINT : PRINT
40  INPUT "ENTER A NUMBER ";N
50 Q =  INT (N / K)
55 C = N - K * Q
60  PRINT "DECIMAL NUMBER","CLOCK NUMBER"
70  PRINT "    ";N,"    ";C
80  PRINT : PRINT : PRINT
85  INPUT "ARE YOU FINISHED ";A$
87  IF  LEFT$ (A$,1) = "Y" THEN 100
90  GOTO 40
100  END
```

```
]RUN
THIS PROGRAM WILL CONVERT A DECIMAL
NUMBER TO A 12-HOUR CLOCK NUMBER.


ENTER A NUMBER 15
DECIMAL NUMBER  CLOCK NUMBER
    15              3



ARE YOU FINISHED RUN
ENTER A NUMBER 89
DECIMAL NUMBER  CLOCK NUMBER
    89              5



ARE YOU FINISHED RUN
ENTER A NUMBER 72
DECIMAL NUMBER  CLOCK NUMBER
    72              0
```

CLOCK AND MODULAR ACTIVITIES

NAME:_____

DATE:_____

CLASS:_____

Directions: Enter the following program and respond to the questions as they appear on the screen. Also, answer the questions below.

Questions:

1. Fill in the following chart using the answers you see on the screen.

BASE #	INTEGER #	CLOCK #

2. Continue the chart above using the values 15, 30, 89 and 230.

3. What was the range of the clock number column in the above chart? _____

4. Now re-enter line 5 as 5 K=8 and repeat questions 1 & 2, making a new chart below:

BASE #	INTEGER #	CLOCK #

5. What is the range of the clock number column in the above chart ?

_____

6. What have you discovered about the relationship between the clock number column and the base column ?

_____

7. Can you predict the answer in the clock number column for the decimal number you enter ?

_____

8. Do you think you can change the program so that it will show the time on a 24-hour clock; or any hour clock ?

General Topic : Graphing

Specific Topic : Computerized Horizontal Bar Graphs

Objective : Students will be able to produce bar graphs using the computer as a tool.

Description : Students will follow directions to enter the number of items in the graph, the label part of the data, the number part of the data, the factor used for division so that the graph fits on the screen, and the symbol choice for the actual display. The computer will produce the graph on the screen.

Uses and Extensions : Carefully type in a copy of the attached program, or use one that has already been saved on a disk. The teacher may want the program entered prior to the day of use due to its length. Note that an Apple IIe should have an 80 column card for this activity.

Sample Items for Student Worksheet

Enter RUN and respond by entering your data or the data provided by your teacher. Ideas for the graph:

Make a graph of the names and ages of everyone in your house. Your factor for doing the division can be 1 unless family members are over 60.

Make a graph listing several members of your class and their scores on an assignment.

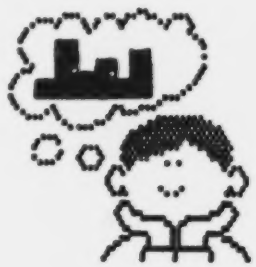
Make a graph comparing the area or the population sizes of the continents (use a copy of the World Almanac for this information).

Make a graph comparing the records of some of the teams that play your favorite sport. (The World Almanac is a good reference).

Listing : Please see next sheet

```
]RUN
HOW MANY DIFFERENT PIECES OF DATA DO YOU HAVE 3
ENTER THE NAME OR IDENTIFIER MICHAEL
ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) 21
ENTER THE NAME OR IDENTIFIER KAREN
ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) 18
ENTER THE NAME OR IDENTIFIER BILL
ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) 35
      A COMPUTERIZED HORIZONTAL BAR GRAPH

      MICHAEL         **********************

      KAREN           ******************

      BILL            ************************************

                      ----+----+----+----+----+----+----+----+----+----
```

```
]LIST

3  REM  THIS WILL RUN ON APPLE IIe WITH AN EXTENDED 80 COLUMN CARD
10  REM  THIS PROGRAM WILL PRINT OUT A HORIZONTAL GRAPH FOR A STUDENT
15  REM  THE MAXIMUM NUMBER OF PIECES OF DATA IS 10
20  REM  TEACHING SUGGESTION, HAVE A STUDENT WORK OUT THE GRAPH ON A PIEC
     E
25  REM  OF GRAPH PAPER AND THEN CHECK HIS OR HER WORK WITH THE COMPUTER
30  REM  N$(X) HOLDS THE NAME PART OF THE DATA
35  REM  S(X) HOLDS THE NUMBER PART OF THE DATA (NO COMMAS)
40  REM  DB IS THE DIVIDING NUMBER
45  REM  T IS THE NUMBER OF SYMBOLS TO BE PRINTED PER LINE
80  DIM N$(10),S(10)
88  HOME
90  PRINT "THIS PROGRAM WILL HELP YOU PRINT A HORIZONTAL BAR GRAPH ON THE
      SCREEN.": PRINT
92  PRINT "YOU WILL NEED TO KNOW THE NAME OR IDENTIFIER AND THE NUMBER TH
     AT"
94  PRINT " GOES WITH THAT INFORMATION. REMEMBER YOU MUST TYPE THE NUMBER
      "
96  PRINT "INTO THE COMPUTER WITHOUT USING ANY COMMAS!!!!!"
100  INPUT "HOW MANY DIFFERENT PIECES OF DATA DO YOU HAVE ";N
110  FOR X = 1 TO N
120  INPUT "ENTER THE NAME OR IDENTIFIER ";N$(X)
130  INPUT "ENTER THE NUMBER (REMEMBER DO NOT USE COMMAS) ";S(X)
140  NEXT X
150  PRINT "THE GRAPH MUST FIT ON THE SCREEN. 60 IS THE MAXIMUM NUMBER"
160  PRINT "OF CHARACTERS THAT CAN BE PRINTED ACROSS THE SCREEN."
170  PRINT "IF THE NUMBERS ARE LARGER THAN 50, THEN YOU HAVE TO DIVIDE AL
     L"
180  PRINT " OF THEM BY THE SAME NUMBER."
190  INPUT "WHAT SYMBOL DO YOU WANT TO USE FOR THE GRAPH ?";SY$
205  HOME
206  PRINT "IF YOUR NUMBERS ARE NOT ABOVE 50, THEN ENTER 1 OTHERWISE ENTE
     R THE NUMBER YOU WISH TO DIVIDE THE NUMBERS BY."
208  INPUT DB
209  PR# 1
210  PRINT "A COMPUTERIZED HORIZONTAL BAR GRAPH": PRINT
220  FOR X = 1 TO N
230  PRINT N$(X),
240  LET T =  INT (S(X) / DB + .5)
250  FOR Y = 1 TO T
260  PRINT SY$;
270  NEXT Y
280  PRINT : PRINT
290  NEXT X
300  PRINT ,"----+----+----+----+----+----+----+----+----+----+----+---"
310  PRINT ,," IN ";DB" UNITS"
315  PR# 0
320  END
```



General Topic : Graphing

Specific Topic : Parabolic Curves

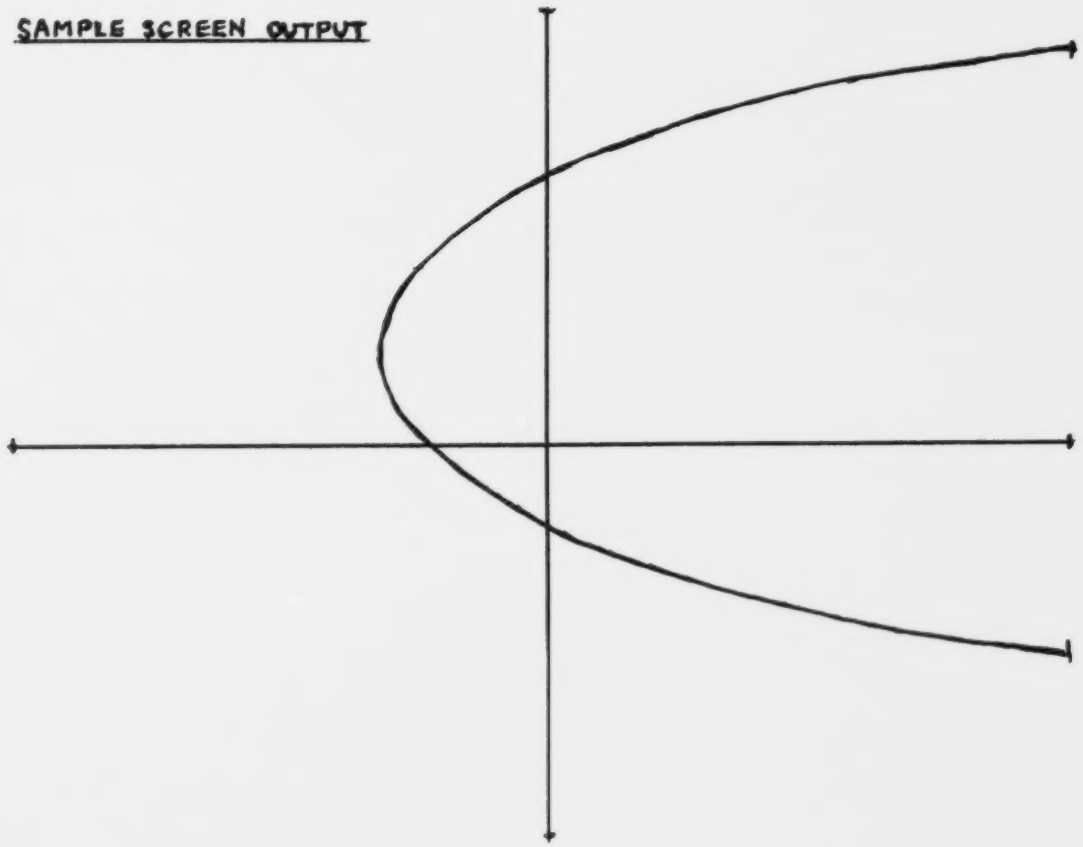
Objective : Students will be able to use a computer to experiment with parabolic equations and to more efficiently explore the beauty of parabolic curves.

Description : The activity uses a computer program to aid students in graphing parabolas of different shapes. A vertical orientation is maintained; axes are included.

Uses and Extensions : Some knowledge of parabolas and their equations is necessary. Worksheets can be designed that guide the student to conclusions about the relationship of coefficients to shape.

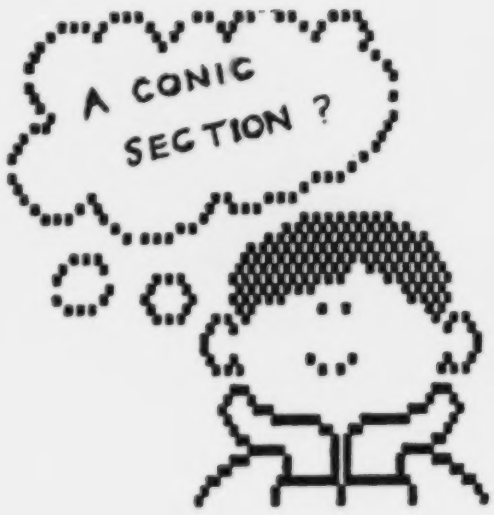
Listing is on the next sheet.

SAMPLE SCREEN OUTPUT



]LIST

```
10  TEXT : HOME
21  PRINT "WHICH EQUATION WOULD YOU LIKE ?": PRINT : PRINT
22  PRINT "     1) X=? * (Y-?) ^2 + ?"
23  PRINT "     2) Y=? * (X-?) ^2 + ?"
24  PRINT : INPUT "       CHOOSE - ";Z$
25 ST = .1
26  IF Z$ = "1" THEN ZY$ = "X":ZX$ = "Y":Z = 1: GOTO 29
27  IF Z$ = "2" THEN ZY$ = "Y":ZX$ = "X":Z = 2: GOTO 29
28  GOTO 10
29  HOME
30  PRINT "INPUT NUMBERS FOR YOUR PARABOLA- ";
40  PRINT : PRINT ZY$;" = ";: INPUT A$
42 LA =  LEN (A$)
45  VTAB 3: HTAB 5
50  PRINT A$;" (";ZX$;" - ";: INPUT B$
52 LB =  LEN (B$)
55  VTAB 3: HTAB 11 + LA
60  PRINT B$;") ^2 + ";: INPUT C$
62 LC =  LEN (C$)
65  VTAB 3: HTAB 18 + LA + LB
70  PRINT C$
80  HGR : HCOLOR= 3: POKE  - 16302,0
100  HPLOT 140,0 TO 140,191
110  HPLOT 0,95 TO 279,95
115  GOSUB 1000
120 A =  VAL (A$):B =  VAL (B$):C =  VAL (C$)
150  IF Z = 1 THEN X =  - 9.5
153  IF Z = 2 THEN X =  - 13.9
160 Y = A * (X - B) ^ 2 + C
165  IF Z = 2 THEN  IF Y > 9.5 OR Y <  - 9.6 THEN X = X + .1: GOTO 160
166  IF Z = 1 THEN  IF Y > 13.9 OR Y <  - 13.9 THEN X = X + .1: GOTO 160
167 Y = ( INT (Y * 10 + .5)) / 10
180  FOR X1 = X TO 0 - X + 2 * B STEP ST
185  IF X1 > 13.5 THEN 230
187  IF X1 > 9.5 AND Z = 1 THEN 230
190 Y = A * (X1 - B) ^ 2 + C
200  IF Z = 2 THEN  IF Y <  - 9.6 OR Y > 9.5 THEN 220
202  IF Z = 1 THEN  IF Y <  - 13.9 OR Y > 13.9 THEN 220
205 Y = ( INT (Y * 10 + .5)) / 10
210  IF Z = 2 THEN  HPLOT 140 + (X1 * 10),95 - (Y * 10): GOSUB 2000
215  IF Z = 1 THEN  HPLOT 140 + (Y * 10),95 - (X1 * 10): GOSUB 2200
220  NEXT X1
230  POKE  - 16301,0: VTAB 23
240  VTAB 23
250  PRINT "WOULD YOU LIKE ANOTHER PARABOLA ?";: GET A$
260  IF A$ <  > "N" AND A$ <  > "Y" THEN  PRINT : HOME : GOTO 240
270  PRINT A$
280  IF A$ = "Y" THEN 10
290  END
1000  FOR S =  - 13 TO 13 STEP 1
1010  HPLOT 140 + S * 10,92 TO 140 + S * 10,98
1020  NEXT S
1030  FOR S =  - 9 TO 9
1040  HPLOT 143,95 - S * 10 TO 137,95 - S * 10
1050  NEXT S
1060  RETURN
2000  HPLOT  TO 140 + (X1 * 10),95 - (Y * 10): RETURN
2200  HPLOT  TO 140 + (Y * 10),95 - (X1 * 10): RETURN
```



General Topic : Graphing

Specific Topic : Graphing elementary algebra functions

Objective : Students will be able to generate tables of values that can then be used in graphing or statistical applications.

Description : The program generates the square, square root, and the reciprocal of the numbers from 1 to K, where K is entered.

Uses and Extensions : After running the program, the student graphs the results. As an extension, modify the program to produce different ranges and functions. The activity can also be used to demonstate interpolation and extrapolation.

```
]LIST

10  HOME
20  PRINT "ENTER THE NUMBER YOU WANT THE TABLE TO GO UP TO. ";: INPUT K
24  PRINT "#";: HTAB (9): PRINT "#^2";: HTAB 19: PRINT "SQR(#)";: HTAB 31
     : PRINT "1/#"
25  FOR X = 1 TO K
26  LET A =  INT (X ^ 2)
27  LET B =  INT (( SQR (X) * 1000)) / 1000
28  LET C =  INT (1 / X * 1000) / 1000
30  PRINT X;: HTAB 10: PRINT A;: HTAB 21: PRINT B;: HTAB 33: PRINT C
35  NEXT X
```

```
]RUN
ENTER THE NUMBER YOU WANT THE TABLE TO GO UP TO. ?7
#       #^2       SQR(#)      1/#
1        1          1          1
2        4          1.414      .5
3        9          1.732      .333
4        16         2          .25
5        25         2.236      .2
6        36         2.449      .166
7        49         2.645      .142


]RUN
ENTER THE NUMBER YOU WANT THE TABLE TO GO UP TO. ?4
#       #^2       SQR(#)      1/#
1        1          1          1
2        4          1.414      .5
3        9          1.732      .333
4        16         2          .25
```

General Topic : Graphing

Specific Topic : Graphing Arbitrary Functions

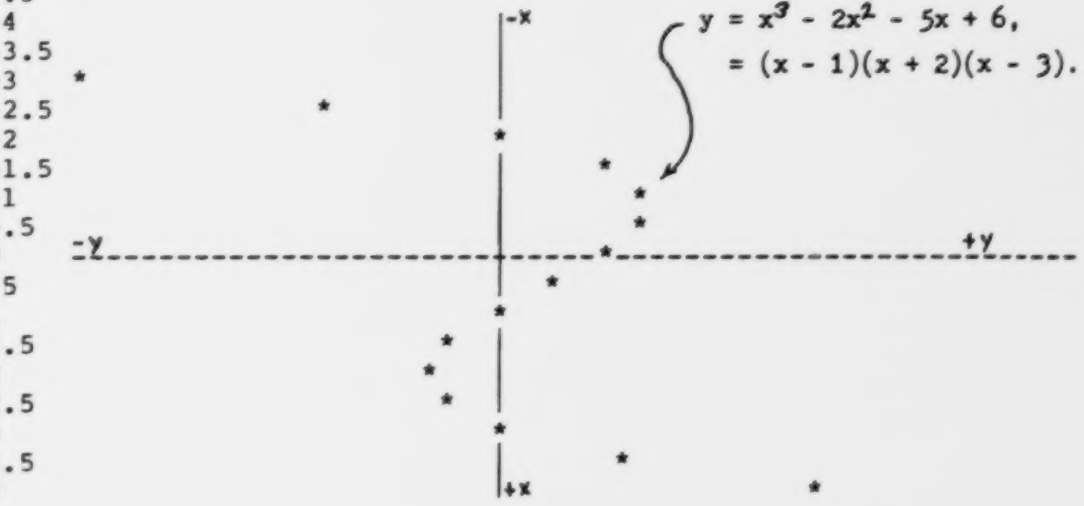
Objective : Students will be able to quickly and efficiently produce graphs of functions.

Description : The activity requires the student to enter a function as part of a computer program. The program then causes the function's graph to be printed. The student must also set (input) the interval and increment (step values of the independent variable) to be used.

Uses and Extensions : Can be used wherever low resolution graphing is needed.
Extensions include refining the step process, introducing the graphing of more than one curve at a time, and estimating the point of intersection of a pair of curves.

Listing on next sheet.

```
 RUN
PLACE YOUR FUNCTION ON LINE 362 IN THE FORM 'Y=F(X)'

HAVE YOU ENTERED A FUNCTION AS YET?


TOUCH '1' FOR YES OR '2' FOR NO
ENTER THE VALUE OF 'X' AT WHICH YOU WISH TO BEGIN.
?-4

NEXT ENTER THE VALUE OF 'X' YOU WISH TO END.
?4

FINALLY, ENTER THE INCREMENT OR DECREMENT.
?.5
```



LIST

```
5  REM  TABLE OF VALUES FOR
6  REM  ANY FUNCTION USER SPECIFIES
10  HOME
20  PRINT "PLACE YOUR FUNCTION ON LINE 362 IN THE FORM 'Y=F(X)'"
30  PRINT
40  PRINT "HAVE YOU ENTERED A FUNCTION AS YET?"
50  PRINT : PRINT
60  PRINT "TOUCH '1' FOR YES OR '2' FOR NO"
70  GET A
80  IF A = 1 THEN 300
90  IF A = 2 THEN  GOTO 190
100  PRINT : PRINT : PRINT "I SAID TOUCH '1' OR '2'"
110  FOR C = 1 TO 1000: NEXT C
120  HOME : GOTO 50
190  HOME : PRINT "OKAY--NOW'S THE TIME TO ENTER THE FUNCTION."
192  PRINT : PRINT : PRINT "ENTER THE FUNCTION ON LINE 200 IN THE FORM 'Y=
     F(X)'"
193  PRINT : PRINT : PRINT "THEN TYPE 'RUN'"
199  END
300  PRINT "ENTER THE VALUE OF 'X' AT WHICH YOU WISH TO BEGIN."
310  INPUT B
320  PRINT : PRINT "NEXT ENTER THE VALUE OF 'X' YOU WISH TO END."
330  INPUT E
340  PRINT : PRINT "FINALLY ENTER THE INCREMENT OR DECREMENT"
350  INPUT I
360  FOR X = B TO E STEP I
362 Y = X ^ 3 - 2 * X ^ 2 - 5 * X + 6
366 Y =  INT (Y + .5)
378  IF  ABS (X) < .00001 THEN X = 0
380  PRINT X;: HTAB (6)
381  IF X = 0 THEN  GOSUB 500: GOTO 390
382  IF  ABS (Y) > 30 THEN  PRINT : GOTO 390
385  PRINT  TAB( Y + 30);"*"
390  NEXT X
400  END
500  FOR L = 1 TO Y + 24: PRINT "-";: NEXT L
510  PRINT "*";: FOR L = Y + 25 TO 60: PRINT "-";: NEXT L
520  PRINT : RETURN
```

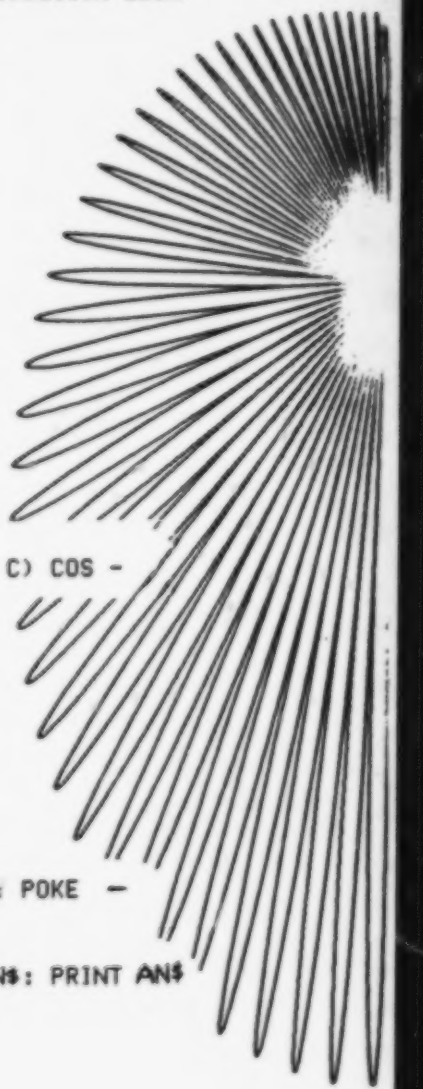
General Topic : Graphing

Specific Topic : Polar Coordinate Graphs

Objective : Students will be able to use a computer to graph and to experiment with polar equations.

Description : The activity centers on a computer program that gives the student choices as to sine or cosine and to the number of multiples (leaves). The program repeats automatically.

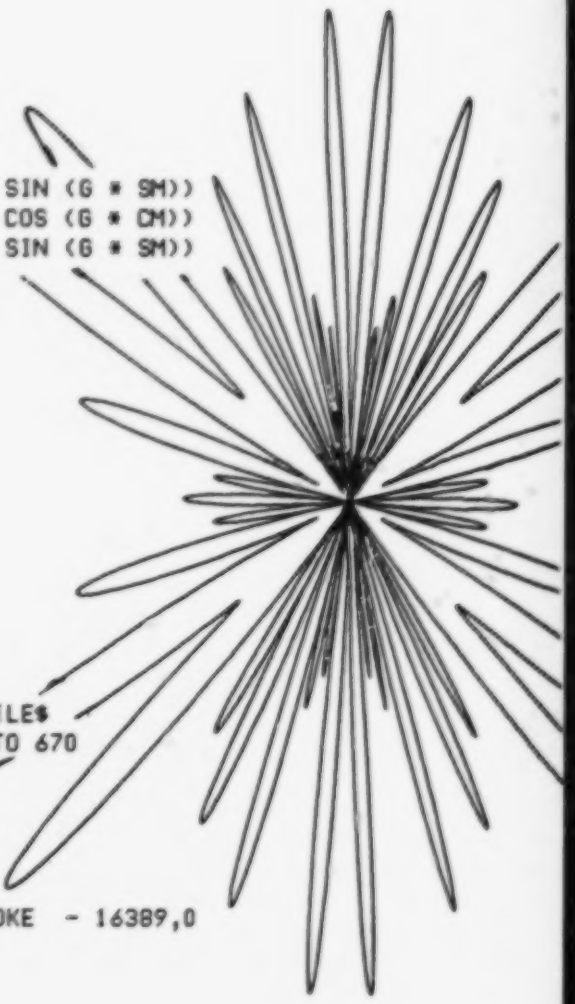
Uses and Extensions : Some knowledge of polar coordinates and equations is necessary. Worksheets can be designed that ask the students to correlate "multiples" to "leaves". Also, by changing lines 120, 140-160, and 500-520, other polar curves and graphs can be explored.
(Text, in line 20, is used with an Apple IIe to reset the screen resolution from graphics to text mode.)



JLIST

```
1  ONERR  GOTO 540
10  GOSUB 540
20  TEXT : HOME
30  INPUT "DO YOU WANT SIN, COS OR BOTH? ";AN$
40  IF  LEFT$ (AN$,1) = "B" THEN  GOSUB 110
50  INPUT "WHAT MULTIPLE DO YOU WANT ";M
55  VTAB 11
60  INPUT "HOW MANY REVOLUTIONS DO YOU WANT ";T
65  INPUT "WHAT SIZE DO YOU WANT (1,2,3) ";SZ
70  INPUT "DO YOU WANT AXIS DRAWN ? ";Y$
75  INPUT "WHAT STEP DO YOU WANT ? ";S
80  PRINT : PRINT  TAB( 3)"REMEMBER- TO STOP PICTURE TYPE 'Q'"
90  PRINT  TAB( 7)"PRESS ANY KEY TO CONTINUE";: GET F$: PRINT
100  GOTO 180
110  HOME
120  PRINT "DO YOU WANT    A) SIN + COS       B) SIN - COS       C) COS -
      SIN ?": INPUT B$
140  INPUT "INPUT THE MULTIPLE FOR THE SIN - ";SM
150  INPUT "INPUT THE MULTIPLE FOR THE COS - ";CM
160  INPUT "INPUT NUMBER TO ADD TO EQUATION";AD
170  RETURN
180  HGR : POKE  - 16302,0: HCOLOR= 3
200 X = 139:Y = 95
210  IF  LEFT$ (Y$,1) = "Y" THEN  GOSUB 470
220 RS = 25
230  GOSUB 320
240  FOR W = 1 TO 1000: NEXT W: POKE  - 16304,0: POKE  - 16297,0: POKE  -
     16301,0
250  POKE  - 16368,0
260  VTAB 23: PRINT "DO YOU WANT TO SAVE THIS PICTURE ?";: GET AN$: PRINT AN$
   .
   .
   .
```

```
270  IF AN$ = "N" THEN 310
280  INPUT "INPUT FILENAME FOR PIC.";FIN$
290  INPUT "WHICH DRIVE IS THE DISK IN ?";D
300  PRINT  CHR$ (4)"BSAVE ";FIN$;",A8192 ,L8191,D";D
310  PRINT : PRINT "PRESS ANY KEY TO RETURN TO MENU";: GET AN$: GOTO 10
320  HPLOT 139,95
330  POKE  - 16368,0
340  FOR G = 0 TO 6.29 * T STEP S
350 Q =  PEEK ( - 16384)
360  IF  LEFT$ (AN$,1) = "C" THEN R1 = SZ *  COS (M * G): GOTO 390
370  IF  LEFT$ (AN$,1) = "B" THEN  GOSUB 500: GOTO 390
380 R1 = SZ *  SIN (M * G)
390 R9 = RS * R1
400 X9 = R9 *  COS (G):Y9 = R9 *  SIN (G)
410  IF (X + X9) ) 279 OR (Y - Y9) ) 191 THEN 450
420  IF (X + X9) ( 0 OR (Y - Y9) ( 0 THEN 450
430  HPLOT  TO X + X9,Y - Y9
440  IF (Q - 128) = 81 THEN 460
450  NEXT G
460  RETURN
470  HPLOT 0,Y TO 279,Y
480  HPLOT X,0 TO X,191
490  RETURN
500  IF B$ = "A" THEN R1 = SZ * (AD +  COS (G * CM) +  SIN (G * SM))
510  IF B$ = "B" THEN R1 = SZ * (AD +  SIN (G * SM) -  COS (G * CM))
520  IF B$ = "C" THEN R1 = SZ * (AD +  COS (G * CM) -  SIN (G * SM))
530  RETURN
540  HOME : TEXT
560  PRINT "DO YOU WANT TO :"
570  PRINT "     1) MAKE A PICTURE"
580  PRINT "     2) SHOW ANOTHER PICTURE
590  PRINT "     3) QUIT"
610  PRINT "           WHICH? ": GET AN$: PRINT AN$
620  IF AN$ = "1" THEN 20
630  IF AN$ = "2" THEN 660
640  IF AN$ = "3" THEN  TEXT : END
655  GOTO 540
660  HOME
665  PRINT "IS THE PICTURE IN MEMORY ?";: GET AN$
666  IF  LEFT$ (AN$,1) = "Y" THEN  PRINT : GOTO 710
667  PRINT
670  INPUT "WHAT IS THE FILENAME OF YOUR PROGRAM   ";FILE$
680  IF FILE$ = "C" THEN  PRINT  CHR$ (4)"CATALOG": GOTO 670
690  IF FILE$ =  CHR$ (13) THEN 540
700  PRINT  CHR$ (4);"BLOAD";FILE$
710  POKE  - 16304,0: POKE  - 16297,0: POKE  - 16300,0
720  POKE  - 16302,0
730  FOR W = 1 TO 1000: NEXT W
740  POKE 16304,0: POKE  - 16297,0: POKE  - 16301,0: POKE  - 16389,0
745  HOME
750  VTAB 22
760  PRINT "   PRESS ANY KEY TO CONTINUE"
```



General Topic : Probability and Statistics

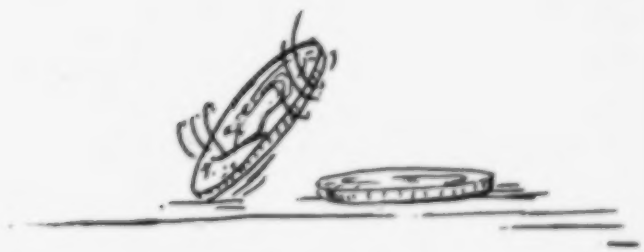
Specific Topic : Coin Toss Simulation

Objective : The student will observe the experimental probability obtained in a simulation of tossing a coin.

Description : This activity simulates tossing a coin and gives the percent of heads and tails.

Uses and Extensions : The program can aid students in learning about probability. As the student enters larger numbers of trials, the law of large numbers will be demonstrated.

Tossing an actual coin can be done and the results tabulated for comparison.



RUN

```
HOW MANY TOSSES DO YOU WISH ?
?10

H T H T T T H H H T

THE COIN WAS TOSSED 10 TIMES

THE NUMBER OF HEADS WAS 5.

THE NUMBER OF TAILS WAS 5.

PERCENT HEADS WAS 50.

PERCENT TAILS WAS 50.
```

]LIST

```
2  HOME
4  PRINT "THIS PROGRAM WILL SIMULATE TOSSING"
6  PRINT "A FAIR COIN ANY NUMBER OF TIMES !"
10  PRINT :K = 1:HC = 0:TC = 0
20  PRINT "HOW MANY TOSSES DO YOU WISH ?"
25  INPUT N: PRINT
30  LET F =  INT ( RND (1) * 2) + 1
40  IF F = 1 THEN D$ = "H":HC = HC + 1
50  IF F = 2 THEN D$ = "T":TC = TC + 1
70  PRINT D$;" ";:K = K + 1
90  IF K <  = N THEN  GOTO 30
100 PH = (HC / N) * 100
110 PT = (TC / N) * 100
115  PRINT : PRINT : PRINT
120  PRINT "THE COIN WAS TOSSED ";N;" TIMES"
130  PRINT : PRINT "THE NUMBER OF HEADS WAS ";HC;".": PRINT
140  PRINT "THE NUMBER OF TAILS WAS ";TC;".": PRINT
150  PRINT "PERCENT HEADS WAS ";PH;".": PRINT
160  PRINT "PERCENT TAILS WAS ";PT;".": PRINT
166  PRINT "DO YOU WANT TO DO THE EXPERIMENT AGAIN": PRINT "ENTER A '1' F
     OR YES AND A '2' FOR NO"
167  INPUT Z: IF Z = 1 THEN  GOTO 2
170  END
```

General Topic : Probability and Statistics

Specific Topic : Generating Frequency Distributions and Their Bar Graphs

Objectives : Students will be able to use a computer to generate a frequency distribution and to display it in bar graph form.

Description : This activity uses a program to generate N (< = 1000) random numbers (integers between 1 and 10 inclusive) and to display the frequency of each in a horizontal bar graph.

Uses and Extensions : This program may be used to investigate the randomness of the computer's random number generator. The graphing portion of this program may be used as is or modified to produce bar graphs for supplied data. For 25 possibilities, add the following lines:

```
105 REM
110 DIM R(25):K=25
120 FOR I=1 TO K: PRINT "FOR POSSIBILITY ";I
130 INPUT "THE FREQUENCY IS "R(I)
```

]LIST

```
100  REM  LINE 130 GENERATES A RANDOM NUMBER AND COUNTS IT
105  INPUT "HOW MANY NUMBERS WOULD YOU LIKE GENERATED ";N
110  PRINT "I AM WORKING-PLEASE WAIT":K = 10
120  FOR I = 1 TO 10:R(I) = 0: NEXT I
130  FOR I = 1 TO N:J =  INT ( RND (1) * 10) + 1:R(J) = R(J) + 1
131  NEXT I
135 B$ = "**************************************"
140  PRINT "NUMBER  FREQUENCY  0--20--40--60--80--100-120-140-160-180"
142  PRINT "------  ---------  +---+---+---+---+---+---+---+---+---+"
150  FOR I = 1 TO K:J =  INT ((R(I) + 2) / 5) + 1
165  PRINT "  ";I;: HTAB 9
166  PRINT R(I);: HTAB (20): PRINT  LEFT$ (B$,J): NEXT I
```

```
]RUN
HOW MANY NUMBERS WOULD YOU LIKE GENERATED? 1000
I AM WORKING-PLEASE WAIT
NUMBER  FREQUENCY  0--20--40--60--80--100-120-140-160-180
------  ---------  +---+---+---+---+---+---+---+---+---+
  1     94         ******************
  2     89         ******************
  3     103        ********************
  4     96         ********************
  5     112        ************************
  6     89         ******************
  7     84         ******************
  8     131        ****************************
  9     104        ********************
  10    98         ********************
```

General Topic : Probability and Statistics

Specific Topic : Playing Cards Simulation

Objective : Students will be able to use the computer to simulate dealing playing cards.

Description : The activity uses a program that:

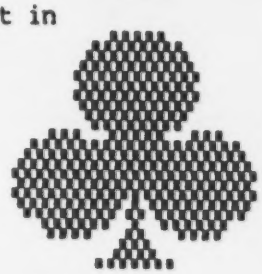
1. Creates a deck of cards, using arrays and strings.

2. Shuffles this deck by using random numbers to swap cards in the array.

3. Deals up to 6 hands, one card at a time, from the top of the deck.

Uses and Extensions :

1. This program may be used to generate empirical data to aid the student in investigating various probabilities as related to card dealing.



2. To generate many poker hands, add the following lines:

```
192 H=6:K=S
194 FOR L=T TO 1000
370 NEXT L
Remove lines 260 and 270
```

3. Many interesting programming assignments, designed to investigate the probability of obtaining various hands, may be given based on this program. For example, to discover how many five card hands must be dealt before a player obtains three of a kind, the following strategy could be employed:

Evaluate the face value of the card by using the remainder of the array subscript divided by 13. Sort these remainders. Check to see if any three of these are equal to each other.

```
]LIST

140  DIM C$(13),D$(52),Z$(10,13):A$(1) = "CLUBS"
150 A$(2) = "SPADES":A$(3) = "HEARTS":A$(4) = "DIAMONDS"
160  FOR I = 1 TO 13: READ C$(I): NEXT I
170  FOR I = 1 TO 4: FOR J = 1 TO 13: REM  CREATE DECK
180 D$((I - 1) * 13 + J) = C$(J) + " OF " + A$(I)
190  NEXT J,I
210  FOR I = 1 TO 52: REM  SHUFFLE
220 J =  INT ( RND (1) * 52) + 1
230 T$ = D$(I):D$(I) = D$(J):D$(J) = T$: NEXT I
260  INPUT "HOW MANY HANDS ";H
270  INPUT "HOW MANY CARDS ";K
275  IF K * H > 52 THEN  PRINT "NOT ENOUGH CARDS": GOTO 260
280  FOR J = 1 TO H: REM  DEAL CARDS
285  PRINT : PRINT "HAND #";J: PRINT "---- --"
290  FOR I = 1 TO K: PRINT D$(I + (J - 1) * K): NEXT I: NEXT J
400  DATA  2,3,4,5,6,7,8,9,10,JACK,QUEEN,KING,ACE
```

```
]RUN
HOW MANY HANDS 2
HOW MANY CARDS 5

HAND #1
---- --
8 OF CLUBS
KING OF DIAMONDS
3 OF DIAMONDS
8 OF SPADES
10 OF CLUBS

HAND #2
---- --
ACE OF HEARTS
QUEEN OF CLUBS
4 OF SPADES
6 OF DIAMONDS
7 OF CLUBS
```

General Topic : Calculus and Advanced Mathematics

Specific Topic : Approximation of Pi by Simulation

Objective : Students will study and experiment with the relationship between the area of a circle and the value of pi.

Description : This activity uses a program that:

1. Draws a unit square with verticies (0,0), (0,1), (1,1), (1,0), and draws a quarter of a unit circle with its center at the origin.
2. Generates and plots 1000 random points within the unit square and counts the number of points which fall on or within the quarter of the circle.
3. Calculates an approximation for the area of the quarter circle, which when multiplied by four, approximates a value of pi since the area of the circle is pi/4. In the program, the variable HITS is used to count the points on or within the quarter circle.

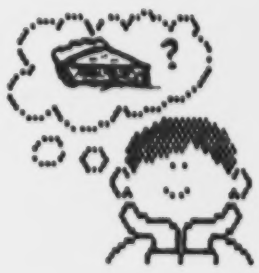
Uses and Extensions : The program may be used to graphically illustrate one method for finding an approximation for pi. By adding the following lines, the number of points generated would be determined by the user of the program:

```
 16 INPUT "HOW MANY POINTS DO YOU WANT DO GENERATE ";N
 19 HOME
120 FOR I=1 TO N
```

The run time of the program will be shortened considerably if the graphics portion of the program is eliminated.

]LIST

```
15  HOME : HGR : HCOLOR= 3:C = 3.14159 / 180
40  HPLOT 230,152 TO 50,152 TO 50,2 TO 230,2 TO 230,152
50  FOR T = 0 TO 90 STEP 10
60 TH = T * C:X =  COS (TH):Y =  SIN (TH)
90  HPLOT  TO X * 180 + 50,152 - Y * 150
100  NEXT T
110  REM  ESTIMATE PI
119  VTAB 23: PRINT "HITS= ";: HTAB 15: PRINT "PI = ";
120  FOR I = 1 TO 1000:A =  RND (1):X = A * 180 + 50
145 B =  RND (10):Y = 152 - B * 150: HPLOT X,Y
170  IF A ^ 2 + B ^ 2 <  = 1 THEN HITS = HITS + 1
180 PI = 4 * (HITS / I)
190  VTAB 23: HTAB 8: PRINT HITS;: HTAB 20: PRINT PI
210  NEXT I
```



General Topic : Computing/Programming

Specific Topic : IRAs and Compound Interest ($$$)

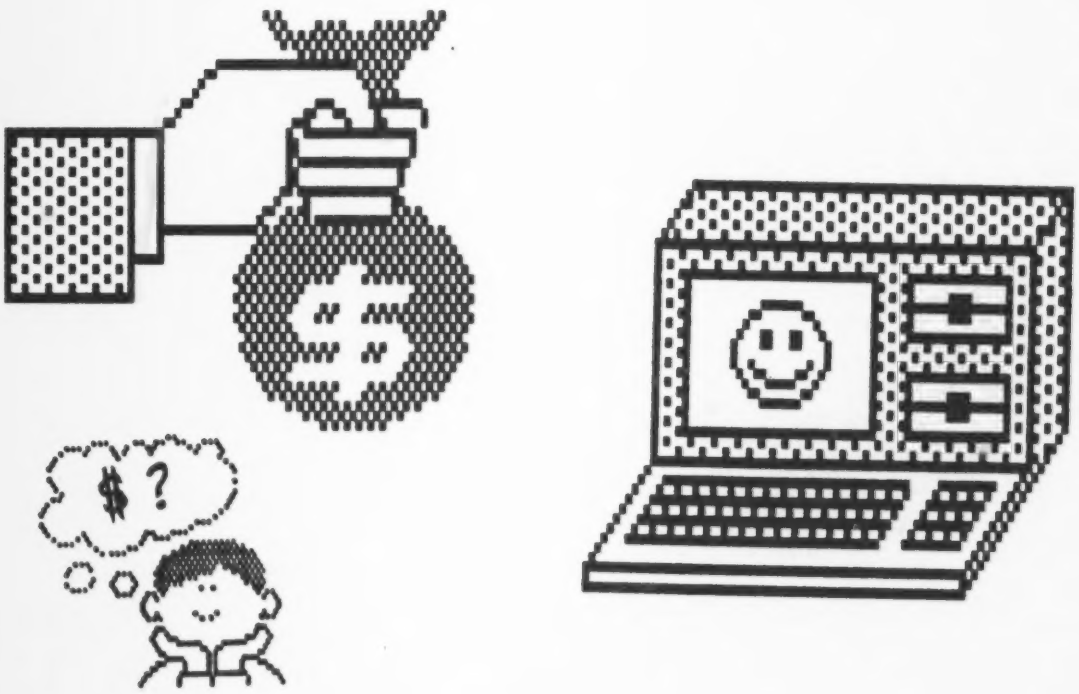
Objectives : Students will be able to use a computer program to produce investment data for graphical or business type exercises.

Description : The activity, through the use of the computer, dramatically demonstrates the value of an Individual Retirement Account. In particular, you are the manager of a bank that has been told to provide IRAs. In order to do so, you design a program that an employee can use to demonstrate dramatically how investing in an IRA today at 8% growth for the life (number of years) of the IRA will mean a large dollar return at retirement. The interest earned is figured on an annual basis using the formula: Interest Earned = Principle * Rate of Growth.

The basic idea is to make an annual calculation, print that result and loop back for another year's growth.

Uses and Extensions : The attached program is typical of what the student programmers may use, or be expected to generate if the activity is used as a programming exercise.

Listing : On next sheet



```
]LIST

212  HOME
215  PRINT "GOOD DAY!!! THE NAME OF THIS PROGRAM IS MAKE MORE MONEY, MMM
     OR $$$ FOR SHORT THIS PROGRAM IS SPONSORED BY THE AMTNJTRUST BANK. I
     T IS DESIGNED TO SHOW ";
217  PRINT "YOU HOW THE INDIVIDUAL RETIREMENT ACCOUNT CAN MAKE YOU MONEY.
      ANYWAY IF YOU WOULD LIKE TO SEE HOW THE DOLLARS CAN BUILD, PLEASE T
     YPE IN YES; IF NOT, TYPE NO."
220  INPUT A$: HOME
221  PRINT "$$$$$$$$$$$$$$$$$$$$$$$$"
222  PRINT "$$$$$$$$$$$$$$$$$$$$$$$$"
225  IF A$ = "NO" THEN 320
230  INPUT "TO BEGIN, WHAT IS YOUR NAME ";B$
233  PRINT "$$$$$$$$$$$$$$$$$$$$$$$$"
234  PRINT "$$$$$$$$$$$$$$$$$$$$$$$$"
235  PRINT "ALSO ";B$;", HOW OLD ARE YOU (YOU MAY LIE IF YOU WISH) ": INPUT
     C
245  LET A = 65 - C
246  PRINT "$$$$$$$$$$$$$$$$$$$$$$$$"
247  PRINT "$$$$$$$$$$$$$$$$$$$$$$$$"
250  PRINT "THAT SHOULD BE ENOUGH PERSONAL INFORMATION ";B$;" BUT I STILL
      NEED SOME MORE INFORMATION. FIRST, I WILL TELL YOU THAT THE INTERES
     T RATE YOU WILL BE USING IS 8%. NOW, ABOUT HOW MUCH MONEY WOULD YOU
     BE PUTTING INTO THE ACCOUNT ";
251  PRINT "MONTHLY? PLEASE TYPE IN YOUR ANSWER.(PLEASE NOTE THAT YOU MAY
      NOT PUT IN MORE THAN $185 MONTHLY)"
255  INPUT M
260  LET M = M * 12: LET R = .08
267  PRINT "YEAR"; TAB( 7);"BEGINNING"; TAB( 18);"YEAR"; TAB( 30);"FINAL"

268  PRINT  TAB( 6);"PRINCIPAL"; TAB( 16);"INTEREST"; TAB( 29);"BALANCE"
270  FOR D = 1 TO A
275  LET P = P + M:I = P * R
286  LET I = I * 100: LET I =  INT (I): LET I = I / 100
290  LET P = P + I
295  PRINT D; TAB( 7);R; TAB( 18);I; TAB( 28);P
300  NEXT D
310  PRINT B$;", AFTER ";D;" YEARS OF PUTTING ";M;" DOLLARS INTO YOUR ACC
     OUNT, YOU WILL END UP WITH ";P;" DOLLARS TO LIVE ON. CONGRATULATIONS
     !! NOW LIFE SHOULDN'T BE AS HARD IF YOU ESTABLISHED AN IRA ACCOUNT N
     OW!!";
313  PRINT "HIT ANY KEY TO CONTINUE ": GET Z$
317  HOME
318  PRINT "WELL, YOU'VE SEEN HOW THE INDIVIDUAL RETIREMENT ACCOUNT FUND
     CAN MAKE YOU MORE MONEY. SO GO GET YOUR OWN ACCOUNT."
320  END
```

General Topic : Computing/Programming

Specific Topic : Computer Generated Solutions (Cupcakes and Quarters)

Objectives : Students will use a computer program to consider the following combinatorial problem:

A customer ordered 15 cupcakes. Cupcakes are placed in packages of 4,3, or 1. In how many ways can you fill the order?

The student (programmer) will need to make several decisions about the task and the resulting program:

1. What is the fundamental strategy for generating a solution list?
2. How efficient is their "search" process?
3. Is it enough to print the list or should the program also summarize by giving a "final answer?"

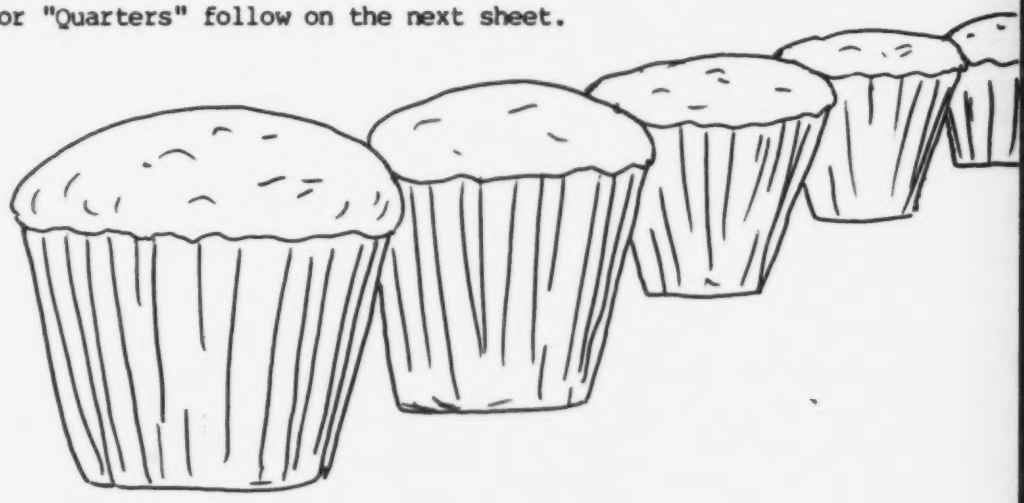
The program and output for "Cupcakes" follow on a separate sheet.

Uses and Extensions : Approaches to this problem will vary with the age and experience of the learner. For example, manipulative materials (to represent the cupcakes) could be used with primary children. Students will be able to see that this problem is typical of the kind of problem where an organized list is required. Hence, this problem is process oriented and not the kind of problem which can be solved by a number sentence and some quick arithmetic.

The objective is clearly not to have elementary and middle school youngsters writing programs of this magnitude. However, it is essential for these learners to explore programs which illustrate the connection between computer use and problem solving. For elementary and middle school students (and also for senior high students), CUPCAKE is an excellent example.

An excellent extension is exemplified by the following: A clerk wants to make change for a quarter and has only dimes, nickels and pennies. In how many ways can the clerk make change for the quarter?

Program and output for "Quarters" follow on the next sheet.



```
]LIST

120  PRINT  TAB( 15);"CUPCAKE PROBLEM": PRINT
150  PRINT "4/PKG","3/PKG"," 1/PKG"
160  PRINT "-----","-----"," -----"
180  FOR J = 0 TO 3: FOR K = 0 TO 5: FOR I = 0 TO 15
210  LET A = J * 4 + K * 3 + I: IF A <  > 15 THEN  GOTO 250
230  LET C = C + 1: PRINT J,"  ";K,"   ";I
250  NEXT I: NEXT K: NEXT J
280  PRINT "THE TOTAL NUMBER OF WAYS IS ";C
```

```
]RUN

                CUPCAKE PROBLEM

4/PKG           3/PKG           1/PKG
-----           -----           -----
0                 0               15
0                 1               12
0                 2               9
0                 3               6
0                 4               3
0                 5               0
1                 0               11
1                 1               8
1                 2               5
1                 3               2
2                 0               7
2                 1               4
2                 2               1
3                 0               3
3                 1               0
THE TOTAL NUMBER OF WAYS IS 15
```

```
RUN

                QUARTER PROBLEM

   HOW MANY WAYS DO YOU THINK THERE ARE ?
        THERE ARE TWELVE WAYS
DIMES           NICKELS           PENNIES
0                 0                 25
0                 1                 20
0                 2                 15
0                 3                 10
0                 4                 5
0                 5                 0
1                 0                 15
1                 1                 10
1                 2                 5
1                 3                 0
2                 0                 5
2                 1                 0
THE TOTAL NUMBER OF WAYS IS 12
```

```
]LIST

1  REM    CHANGE OF A QUARTER
10  INPUT "HOW MANY WAYS DO YOU THINK THERE ARE TO MAKE CHANGE FOR A QUAR
     TER? ";A
15  IF A <  > 12 THEN  PRINT "SORRY, THERE ARE TWELVE WAYS-LETS SEE THEM!
     !": FOR X = 1 TO 1000: NEXT X: GOTO 25
20  IF A = 12 THEN  PRINT "VERY GOOD!!! LET'S SEE THEM": FOR X = 1 TO 100
     0: NEXT X
25  PRINT "DIMES          NICKELS             PENNIES"
30  FOR D = 0 TO 2: FOR N = 0 TO 5: FOR P = 0 TO 25
40  LET X = (D * 10) + (N * 5) + P
50  IF X <  > 25 THEN  GOTO 100
60 C = C + 1
70  PRINT D,N,P
100  NEXT P: NEXT N: NEXT D
130  PRINT "THE TOTAL NUMBER OF WAYS IS ";C
```

General Topic : Calculus and Advanced Mathematics

Specific Topic : Frequently Used Maclaurin Series for sin(x), cos(x), and arctan(x).

Objective : The students will be able to use a computer to determine values of the cos(x), sin(x), and arctan(x) by using a power series.

Description : The activity uses three simular programs based on power series. The student will specify the number of terms of the series to be used.

The first program uses the series

$$\sin(x) = X - (X^3)/3! + (X^5)/5! - (X^7)/7! + \ldots$$

to approximate the value of the sine for X in radian measure. An overflow error may result if more than 17 terms are specified.

The second program uses the series

$$\cos(x) = 1 - (X^2)/2! + (X^4)/4! - (X^6)/6! + \ldots$$

to approximate the value of the cosine for radian measure.

The third program uses the Taylor Series to evaluate the arctan(x), provided $-1 < X < 1$; that is,

$$\arctan(x) = X - (1/3)X^3 + (1/5)X^5 - (1/7)X^7 + \ldots.$$

Uses and Extensions : A sample worksheet is attached. The students also may want to compare their results with the built-in BASIC functions of SIN(x), COS(x) and ATN(x).

Sample questions may include:

1. After what number of terms in each series did the approximation obtained give the same result as the library function?
2. Do these series converge rapidly or slowly?

As an extension, challenge students to determine a formula for tan(x) using the series in the first and second programs and check their results using the formula for the arctan(x). For example, they can determine the tan(1) using sin(1) and cos(1) and check whether the arctan of this value is equal to 1.

→

WORKSHEET

1. Enter the following program:

```
 5 REM***EVALUATE SIN(X) BY A SERIES**
 7 PRINT CHR$(147)
10 INPUT"ENTER THE VALUE OF X IN RADIANS";X
20 S=X:D=1:K=1
35 INPUT"HOW MANY TERMS DO YOU WISH TO ADD";W
40 FOR I= 3 TO 2*W STEP 2
50 D=D*(I-1)*I:K= -K
60 S=S+K*X^I/D:PRINT S
80 NEXT I
90 PRINT"APPROXIMATION OF SINE(X) USING ";W;" TERMS IS ";S
```

2. Determine the sin(0.5) using 5 terms._____ using 10 terms._____ using 15 terms._____

3. Use the program to determine the best approximation of the sin(x) if x=

   (a) .523598 radians _____

   (b) .785397 radians _____

   (c) 1.0472 radians _____

   (d) 1 radian _____

   (e) 30 degrees _____

   (f) 90 degrees _____

4. Make the following changes in the program above:

   (a) Change SIN(X) to COS(X) in lines 5 and 90
   (b) Change line 20 to S=1:D=1:K=1
   (c) Change line 40 to FOR I=2 TO S*W-2 STEP 2

   Now, determine the cos(0.5) using 5 terms._____ using 10 terms._____ using 15 terms._____

5. Use this program to determine the best approximation of the cos(x) if x=

   (a) .523598 radians _____

   (b) .785397 radians _____

   (c) 1.0472 radians _____

   (d) 1 radian _____

   (e) 30 degrees _____

   (f) 90 degrees _____

6. Enter the following program:

```
 5 REM***EVALUATE ARCTAN USING A SERIES***
 7 PRINT CHR$(147)
10 INPUT"ENTER A VALUE BETWEEN 1 AND -1 FOR X";X
15 IF X<-1 OR X>1 THEN PRINT "VALUE OUT OF RANGE":GO TO 10
20 S=0
25 INPUT "HOW MANY TERMS DO YOU WISH TO USE ";W
30 FOR I=1 TO 2*W-3 STEP 4
40 S=S+X^I-X^(I+2)/(I+2):PRINT S
60 NEXT I
70 PRINT"APPROXIMATION OF ARCTAN(X) USING ";W;" TERMS IS ";S;" RADIANS"
```

7. Determine the arctan(0.5) using 5 terms._____ using 10 terms._____ using 15 terms._____

8. Determine the arctan(0.34) using 8 terms._____

9. Determine the best approximation for the arctan(x) if x is:

(a) 0.5 _____

(b) 0.25 _____

(c) -0.5 _____

(d) -0.75 _____

(e) 0.75 _____

(f) 0.1 _____

(g) 0 _____

(h) 2 _____

(i) -3 _____

10. (optional)

Can you use the first and second programs to determine the tan(x), where x is 1 radian, and check this result with the arctangent program ?

General Topic : Calculus and Advanced Mathematics

Specific Topic : Using Infinite Series to Approximate Pi and e.

Objective : The student will use a power series and the computer to approximate pi and e.

Description : The activity uses two series to approximate pi and e. The series used in the first program to approximate pi is:

$$pi = 4 * (1- (1/3) + (1/5) - (1/7) + (1/9) - (1/11) + ...)$$

This series does not converge quickly, so students will have to select a large number of terms for their approximation of pi.

The series to approximate e is:

$$e = 1 + (1/1!) + (1/2!) + (1/3!) + (1/4!) + ...$$

It is a specific case of the more general formula:

$$e^x = 1 + (x/1!) + (x^2/2!) + (x^3/3!) + (x^4/4!) + ...$$

This series converges much more rapidly than the one selected to approximate pi. An overflow error will result if the students select more than 34 terms in the series.

Uses and Extensions : A sample worksheet is attached. The formula used to determine pi was used by the German mathematician Gottfried Leibniz (1646-1716). It is not as useful for computing the value of pi as some other series because it converges so slowly.

Extensions include having students research and explore other series which approximate pi more rapidly, emphasizing the idea of convergence. For example:

$$pi = 2 * 1 + \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1 * 3}{2 * 4}\left(\frac{1}{5}\right) + \frac{1 * 3 * 5}{2 * 4 * 6}\left(\frac{1}{7}\right) + \frac{1 * 3 * 5 * 7}{2 * 4 * 6 * 8}\left(\frac{1}{9}\right) + \ldots ,$$

or

$$pi = 6 * \frac{1}{SQR(3)}\left\{1 - \frac{1}{3*3} + \frac{1}{3^2*5} - \frac{1}{3^3*7} + \frac{1}{3^4*9} - \ldots\right\}.$$

You may also wish to have the students determine the value of e raised to a power. As a programming exercise, you may ask students to write their own programs for using the above series.

Sample Work Sheet

1. Enter the following program:

```
  5 REM ***EVALUATE PI USING A SERIES***
  7 PRINT CHR$(147)
 10 I=1:S=4:K=1
 30 INPUT"HOW MANY TERMS DO YOU WISH TO USE ";W
 40 FOR I=2 TO W
 50 K= -K: S = S+K*4/(2*I-1): PRINT S
 80 NEXT I
 90 PRINT"APPROXIMATION OF PI AFTER ";W;" TERMS IS ";S
```

2. Use the program to evaluate pi after:

   5 terms _____ 10 terms _____ 50 terms _____

   100 terms _____ 200 terms _____ 500 terms _____

3. The best approximation of pi to 6 decimal places occurs after using how many terms in the series ? _____

4. Explain whether this is a slowly or rapidly converging series.

5. Enter the following program:

```
  5 REM***EVALUATE E USING A SERIES***
  7 PRINT CHR$(147)
 10 S=1:P=1
 30 INPUT"HOW MANY TERMS DO YOU WISH TO ADD ";W
 40 FOR I=1 TO W-1
 50 P=P*I:S=S+1^I/P:PRINT S
 80 NEXT I
 90 PRINT"APPROXIMATION OF E USING ";W;" TERMS IS ";S
```

6. Evaluate e after:

   5 terms _____ 10 terms _____ 20 terms _____

   25 terms _____ 30 terms _____

7. The best approximation occurs after how many terms? _____

8. Is this a slowly or rapidly converging series ? _____

General Topic : Calculus and Advanced Mathematics

Specific Toopic : The Solution of Equations of Any Degree for Real Roots

Objective : Students will be able to use a method by which they can find the real roots, both rational and irrational, of an equation of any degree in one variable to any reasonable degree of accuracy without resorting to the traditional methods of elementary algebra.

Description : The activity uses a program, named TABLE OF VALUES, to find the real roots of an equation of any degree in one variable to any degree of accuracy desired, through the nearest thousandth, by the location principle.

Uses and Extensions : After typing in the program TABLE OF VALUES:

1. Decide on the maximum number of real roots the equation can have.
2. Set the equation equal to zero, replace the zero with "Y" (thus obtaining the corresponding function), run the program TABLE OF VALUES on the computer and enter the function as instructed in the program.
3. Enter the starting and ending points for the table with the increment of 1. The starting and ending points are up to you. The computer will now respond by generating the "X" and "Y" values between the points you designated as starting and ending points. It will prove valuable to have the students make a graph of the ordered pairs generated. You can then discuss the relationship between the zeros of the function just graphed and the roots of the corresponding equation.
4. Now it is time to search for the sign changes in the "Y" value and find the corresponding "X" values as the new starting and ending points as you run the program again. Of course this process will have to be repeated several times until you can read the root to the nearest thousandth. See the print-out ot the "run" of the program as an illustration. For this sample "run", we'll use:

$$Y = 4*X^5 - 4*X^4 - 15*X^3 - 6*X^2 + 2*X + 1$$

BE SURE YOU ENTER THE FUNCTION ON LINE 200 AS IT APPEARS HERE AND IN THE PROGRAM LISTING.

A. We expect a maximum of five real roots since our function is of fifth degree. First we'll run the table for "X" between -10 and 10 with an increment of 1. From the printout of the "run", it's clear that there is a real root at -1, another between 0 and 1 and still another between 2 and 3 because of the "Y" value of 0 in the case of -1 and the sign changes in "Y" between 0 and 1 and between 2 and 3. We'll concentrate on the root between 0 and 1.

NOTE - The teacher is cautioned that at times, due to round-off, the computer will produce a "Y" value in scientific notation with an exponent of -08 or -09. In all probabilty, the corresponding "X" value is a real root of the equation.

B. Now we'll reRUN and begin at 0 and end at 1 with an increment of .1. The printout clearly shows a sign change in "Y" between .3 and .4.

C. Next, we'll reRUN and begin at .3 and end with .4 with an increment of .01. This time we observe the sign change in "Y" between .38 and .39.

D. Now start at .38 and end at .39 with increment .001. The sign change in "Y" is between .381 and .382.

E. Finally, we reRUN and start at .381 and end at .382. This time, due to round-off, the sign change in "Y" is between .381899999 and .381999999. Clearly, the real root to the nearest thousandth is .382.

The results seen here obviouly lead into a discussion of rational and irrational

roots. It is important for students to see that equations of any degree in one variable can be solved for their real roots via this method. It also "opens the door" to a discussion of the graphs of functions, their relation to the roots of corresponding equations when the function value is zero, and can lead directly to the solution of maximum-minimum problems using only the TABLE OF VALUES program and no Calculus.

]LIST

```
5  HOME : REM  TABLE OF VALUES FOR ANY FUNCTION USER SPECIFIES
20  PRINT "PLACE YOUR FUNCTION ON LINE 200 IN THE FORM 'Y=F(X)'"
40  PRINT "HAVE YOU ENTERED A FUNCTION YET ?": PRINT : PRINT
60  INPUT "INPUT 1 FOR YES OR 2 FOR NO ";A
80  ON A GOTO 300,190
100  PRINT : PRINT "I SAID ENTER A 1 OR A 2"
110  FOR C = 1 TO 1000: NEXT C: HOME : GOTO 60
190  HOME : PRINT "OKAY--NOW'S THE TIME TO ENTER THE FUNCTION."
192  PRINT "ENTER THE FUNCTION ON LINE 200 IN THE FORM Y= F(X)."
193  PRINT : PRINT : PRINT "THEN TYPE 'RUN'": END
200 Y = 4 * X ^ 5 - 4 * X ^ 4 - 15 * X ^ 3 - 6 * X ^ 2 + 2 * X + 1
202  IF Y > 100 THEN Y = Y * 1000
203  IF Y > 100 THEN Y =  INT (Y)
204  IF Y > 100 THEN Y = Y / 1000
210  RETURN
300  INPUT "ENTER THE VALUE OF X AT WHICH YOU WISH TO BEGIN.";B
320  INPUT "ENTER THE VALUE OF X AT WHICH YOU WISH TO END.";E
340  INPUT "FINALLY, ENTER THE INCREMENT OR DECREMENT";I
355  HOME : PRINT : PRINT : PRINT "       X","       F(X)"
357  PRINT "    -------","    -------"
360  FOR X = B TO E STEP I
370  GOSUB 200
372 M = 100000 * X:M =  INT (M)
376  IF M =  - 1 THEN X = 0
377 N = 100000 * Y:N =  INT (N)
379  IF N =  - 1 THEN Y = 0
380  PRINT  TAB( 5);X;
381  HTAB 20: PRINT Y: FOR K = 1 TO 50: NEXT K
390  NEXT X
```

1 ST RUN

```
ENTER THE VALUE OF X AT WHICH YOU WISH TO BEGIN.0
ENTER THE VALUE OF X AT WHICH YOU WISH TO END.1
FINALLY, ENTER THE INCREMENT OR DECREMENT.1

       X              F(X)
    -------        -------
    0              1
    .1             1.12464
    .2             1.03488
→   .3             .63232      } NOTE SIGN CHANGE
→   .4             -.181440003 }
    .5             -1.5
    .6             -3.40736
    .7             -5.97312001
    .8             -9.24768002
    .9             -13.25744
```

```
INPUT 1 FOR YES OR 2 FOR NO 1
ENTER THE VALUE OF X AT WHICH YOU WISH TO BEGIN.(.3)
ENTER THE VALUE OF X AT WHICH YOU WISH TO END.(.4)
FINALLY. ENTER THE INCREMENT OR DECREMENT.01
```

2ND RUN

X	F(X)
.3	.63232
.31	.57104582
.32	.505558732
.33	.435762316
.34	.361560728
.35	.282858749
.36	.199561829
.37	.111576142
.38	.0188086266 }
.39	-.0788329621 }

3RD RUN

X	F(X)
.38	.0188086263
.381	9.26531618E-03 }
.382	-3.26829962E-04 }
.383	-9.96789802E-03
.384	-.0196579853
.385	-.0293971784
.386	-.0391855733
.387	-.0490232552
.388	-.0589103233
.389	-.0688468614

4TH RUN

X	F(X)
.381	9.26531595E-03
.3811	8.30830214E-03
.3812	7.350798E-03
.3813	6.39280677E-03
.3814	5.43432566E-03
.3815	4.47535748E-03
.3816	3.51589802E-03
.3817	2.55595287E-03
.3818	1.59551553E-03
.381899999	6.34592492E-04 }
.381999999	-3.26822978E-04 }

.3819 is the root, correct to four places.

General Topic : Calculus and Advanced Mathematics

Specific Topic : Newton's Method for Approximating Roots

Objectives : Students will be able to approximate the roots of an equation by use of a computer.

Description : The activity uses Newton's Method. Thus, the students have to enter the function, it's derivative, and an initial estimate. Directions are included in the program.

Uses and Extensions : The activity may be used with worksheets or to lead to discussions of roots, degrees, and the graphs of functions.

```
]LIST

1  HOME : REM  ROOT FINDING BY NEWTON'S METHOD
3  REM  TYPE 'RUN 10' TO SKIP INTRO
7  PRINT "ON LINE 40 ENTER THE FUNCTION IN THE 'FX=    '.": PRINT "ON LINE
     45,ENTER THE DERIVATIVE IN THE FORM 'DF=    '"
8  FOR J = 1 TO 5000: NEXT J: HOME : VTAB 12
10  PRINT "ENTER YOUR INITIAL ESTIMATE ";: INPUT X
20  PRINT "HOW MANY TRIALS DO YOU WISH ";: INPUT K
30  FOR I = 1 TO K
40 FX = 3 * X ^ 4 + X ^ 2 - 1
45 DF = 12 * X ^ 3 + 2 * X
48  IF DF = 0 THEN  PRINT "THE DERIVATIVE IS ZERO FOR YOUR ESTIMATE. RERU
    N WITH A NEW ESTIMATE.": GOTO 10
50 NX = X - FX / DF
55  PRINT "ESTIMATE NUMBER ";I;" IS ";NX:X = NX
67  FOR TI = 1 TO 500: NEXT TI: NEXT I
```

```
]RUN
ON LINE 40 ENTER THE FUNCTION IN THE 'FX=    '.
ON LINE 45,ENTER THE DERIVATIVE IN THE FORM 'DF=    '
ENTER YOUR INITIAL ESTIMATE ?-6
HOW MANY TRIALS DO YOU WISH ?8
ESTIMATE NUMBER 1 IS -4.49347158
ESTIMATE NUMBER 2 IS -3.36181785
ESTIMATE NUMBER 3 IS -2.51131084
ESTIMATE NUMBER 4 IS -1.87244483
ESTIMATE NUMBER 5 IS -1.39520865
ESTIMATE NUMBER 6 IS -1.04716102
ESTIMATE NUMBER 7 IS -.813828784
ESTIMATE NUMBER 8 IS -.69298731
```

General Topic : Advanced Math and Calculus

Specific Topic : Numerical integration using the Monte Carlo method

Objective : Students will be able to use the Monte Carlo method as an additional method to evaluate a definite integral, particularly one related to the area under a curve.

Description : Students will enter the lefthand and righthand endpoints of a chosen interval. They will also enter the minimum and maximum values of the function on that interval. The computer will find the area of the (lower) rectangle formed by the minimum functional value (height) and the length of the interval (base). The computer will then "throw" darts into the upper rectangle and find the proportion of that area that is under the curve by computing the ratio of "hits" to the total thrown. Adding these two areas represents the area under the curve.

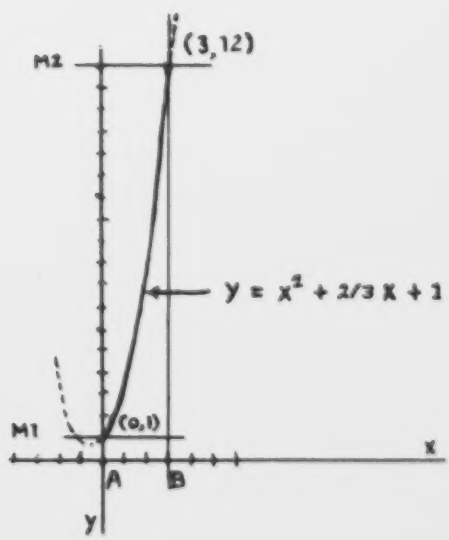
Uses and Extensions : Enter the program and type RUN. Respond by inputting your data or that provided by your teacher.

Make a table comparing the area found to the number of trials used. As you increase the number of trials, what happens? Find the exact area by integration and compare this to values the computer found for the area, particularly where the number of trials was large. How many trials would it take to find the exact area under the curve?

Change line 200 to:

```
200 DEF FNF(X)=SQR(2-X^2)   or to   200 DEF FNF(X)= SIN(X)
```

Choose the endpoints of an interval and the minimum and maximum values of the function on that interval.
Enter a fairly large number for the number of trials. Does the result surprise you?



A sample run based on the figure at the left follows on the next sheet.

Interval: A = Ø B = 3

Minimum: M1 = 1 (for A = Ø)

Maximum: M2 = 12 (for B = 3)

(Note: the answer found by using integration is 15.)

```
]LIST

190  REM   DEFINE FUNCTION IN LINE 200
200  DEF  FN F(X) = X ^ 2 + 2 / 3 * X + 1
240  PRINT "INPUT ENDPOINTS OF THE INTERVAL ON WHICH YOU ARE INTEGRATING"

250  INPUT "A AND B ";A,B
290  PRINT "INPUT MIN AND MAX VALUE OF THE FUNCTION ON INTERVAL (A,B)"
300  INPUT "(MIN AND MAX) ";M1,M2
340  REM  THE AREA OF THE LOWER RECTANGLE
350 Z1 = (B - A) * M1:RS = 0
405  PRINT "INPUT THE NUMBER OF TRIALS YOU WANT---LARGE NUMBERS TAKE A LO
     NG TIME"
450  INPUT "NUMBER OF TRIALS ";N
490  REM   THROW DARTS IN THE UPPER RECTANGLE AND COUNT HOW MANY ARE UNDE
     R OR ON THE CURVE
500  FOR I = 1 TO N
550 X1 = A +  RND (I) * (B - A)
600 Y1 = M1 +  RND (I) * (M2 - M1)
650  IF Y1 < = FN F(X1) THEN S = S + 1
700  IF  INT (I / 100) = I / 100 THEN  GOSUB 1100
750  NEXT I
790  REM  CALCULATES THE AREA UNDER THE CURVE AS THE RATIO OF THE NUMBER
     OF DARTS THROWN UNDER THE CURVE TO THE TOTAL THROWN
800 Z2 = (S / (I - 1)) * (B - A) * (M2 - M1)
840  REM  AREA UNDER THE CURVE IS THE SUM OF THE LOWER RECTANGLE AND THE
     PORTION OF THE UPPER RECTANGLE
850 Z = Z1 + Z2: VTAB 10: PRINT : REM  OUTPUTS AREA UNDER THE CURVE
950  INVERSE : PRINT "THE AREA UNDER THE CURVE IS ";Z
1000  NORMAL : END
1100  VTAB 10: HTAB 17: PRINT "WORKING": RETURN
```

```
]RUN
INPUT ENDPOINTS OF THE INTERVAL ON WHICH YOU ARE INTEGRATING
A AND B 0,3
INPUT MIN AND MAX VALUE OF THE FUNCTION ON INTERVAL (A,B)
(MIN AND MAX) 1,12
INPUT THE NUMBER OF TRIALS YOU WANT---LARGE NUMBERS TAKE A LONG TIME
NUMBER OF TRIALS 100
                WORKING

THE AREA UNDER THE CURVE IS 13.23

      ]RUN
      INPUT ENDPOINTS OF THE INTERVAL ON WHICH YOU ARE INTEGRATING
      A AND B 0,3
      INPUT MIN AND MAX VALUE OF THE FUNCTION ON INTERVAL (A,B)
      (MIN AND MAX) 1,12
      INPUT THE NUMBER OF TRIALS YOU WANT---LARGE NUMBERS TAKE A LONG TIME
      NUMBER OF TRIALS 100
                      WORKING

      THE AREA UNDER THE CURVE IS 15.87
```

PROGRAM CONVERSION SUGGESTIONS

BASIC instructions, such as READ, PRINT, LET, INPUT, FOR/NEXT, IF/THEN, ON N GOTO, GOTO, etc., that are common to most brands of microcomputers were omitted from the following table.

Apple IIe	TRS-80	Commodore
INT(N*RND(1) + 1)	RND(N)	INT(N*RND(1) + 1)
GET A$	INPUT A$	INPUT A$
HOME	CLS	<CLEAR HOME>
HTAB(N)	TAB(N)	TAB(N)
VTAB(N)	FOR I=1TO N:?:NEXT I	FOR I=1TO N:PRINT:NEXT I
^ (exponentiation)	↑	^
<ESC>-<G>	<BREAK>	<BREAK>
PR#1 PRINT exp,... PR#0	LPRINT exp,...	OPEN 3,4: CMD 3 PRINT #3, exp,... CLOSE 3

Apple: 10 IF rel. THEN GOSUB 1n1: GOTO 1n3
20 GOSUB 1n2

Commodore: same as above

TRS - 80: 10 IF rel. THEN GOSUB 1n1 ELSE GOSUB 1n2

DISK COPIES

Disk copies of the programs contained in the Monograph are under consideration for the Apple IIe, TRS-80 III & IV, and Commodore. For further information, contact Dr. Fran Masat at Glassboro State College, Glassboro, NJ 08028, or at (609) 863-6045.

SOFTWARE REFERENCES

The following pages list software that is (or may be) useful for mathematics instruction. Any such listing of materials must be accompanied by some caveats. First, the information goes out of date very quickly. Second, the list is uneven; you won't find any quality judgments. Third, the listing suffers from the exigencies of trying to condense information into a small space.

The abbreviations IM, SM, TB stand for Instructor's Manual, Student Manual, and Textbook, respectively.

TITLE MODE	AUTHOR PUBLISHER, DATE	LEVEL/COURSE	IM	SM	TB	DISK
COMPUCALC:Computer Calculus; Supplementary exercises, Classroom demo's;	R.Weaver Worth, 1984	Calculus I,II	N	Y	Y	1
Calc. for Scientists/Egrs; Numerical Calculations;	Scientific software Inc. Wiley, 1984	Calculus I, II, III	Y	N	N	1
Calculus Demonstrations: Instructor demonstrations, Graphics;	J. Mahoney Sidwell Friends, 1980	Calculus	N	N	N	1
Calculus Stdnt's Toolkit; Supplement, Numerical & graphics prgms.;	R. Finny & D. Hoffman Addison-Wesley, 1984	Calculus I, II, III	Y	Y	Y	4
Comp Test Bank; Test Construction, Calc. book supplement;	J. & A. Kleinstein Saunders Coll., 1984	Calculus I, II, III	Y	N	N	?
Computer Graphing Expers.; Guided Tutorial, Workbook exercises;	C.Lund & E. Anderson Addison-Wesley, 1982	Pre-calculus	Y	Y	N	3
Computerized Test Bank; Test Construction, Interm. Alg. supplement;	J. & A. Kleinstein Saunders Coll., 1983	Pre-calculus	Y	N	N	2
Enns Algebra Package; Instructor demonstrations, Supplemetary;	D.K. Enns & W. Dodge D.C. Health, 1984	Pre-calculus	Y	N	N	1
Enns Calculus Pkg; Instructor demonstrations, Supplementary;	D.K. Enns & W. Dodge D.C. Health, 1984	Calculus I, II	Y	N	N	1
Enns College Math Pkg; Instructor demonstrations, Supplementary;	D.K. Enns & W. Dodge D.C. Health, 1984	Calculus I, II	Y	N	N	1
Enns Jr. High Math Pkg; Arith., Alg., Geometry drill games;	D.K. Enns & W. Dodge D.C. Health, 1984	Pre-college	Y	N	N	1
Enns Math w/Applications; Instructor demonstrations, Supplementary;	D.K. Enns & W. Dodge D.C. Health, 1984	Pre-calculus, Calculus I	Y	N	N	1

TITLE MODE	AUTHOR PUBLISHER, DATE	LEVEL/COURSE	IM	SM	TB	DISK
Enns Secondary Math Pkg; Arith., Alg., Geometry instructor demo's;	D.K. Enns & W. Dodge D.C. Health, 1984	Pre-college	Y	N	N	1
Enns Super Math Pkg; Contains all Enns pkgs;	D.K. Enns & W. Dodge D.C. Health, 1984	Varied	Y	N	N	2
Grapforth; Supplemental;	--- Insoft, 1984	Geometry	Y	N	N	1
Int. Experiments/Calculus; Tutorial, Experimental;	M. & F. Wattenberg Prentice-Hall, 1984	Calculus I, II, III	N	Y	N	1
Interface:Calc & Computer; Supplementary textbook, Graphics programs;	D. Smith Saunders Coll.; 1984	Calculus I, II	Y	N	Y	1
Linear Algebra; Supplement; Numerical & graphics prgms;	G. & D. Williams Allyn/Bacon, 1984	Postcalculus	N	Y	Y	1
Math CAI Programs; Drill and practice;	H. Africk & H. Braverman N.Y.C. TECH, 1983	Pre-calculus	N	Y	N	2
MATH GRAPH; Supplementary, Numerical & graphics prgms;	E. Gade & J. Oman Macmillan, 1983	Calculus I,II	Y	N	N	1
Math Worlds; Supplementary, simulations, worksheets;	--- Stirling Swift, 1984	Middle School Math	Y	N	N	6
Matrices, Determinants, and Simultaneous Equ.; Supplementary, generates problems;	--- Radio Shack, 1984	Advanced Math	Y	N	N	1
Microcomputer Graphics; Textbook on graphics;	R. Myers Addison-Wesley, 1982	Popular	N	N	Y	1
Plane Analytic Geometry; Supplementary, lines and conic sections;	--- Radio Shack, 1984	Calculus I, II, III	Y	N	N	1
Statistical Analysis; Statistical Analysis;	J. Russ Sci. MicroProgs, 1980	Statistics	Y	N	N	1
The Kings Rule; Functions, logical thinking;	T. O'Brien Sunburst, 1983	Pre-Algebra, Algebra	Y	N	N	1
muMATH/muSIMP-80 (ADIOS); Symbolic and High precision calculations;	Soft Warehouse Microsoft, 1981	Calculus I, II, III+	Y	N	N	3